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1 Introduction

After the quantum chromodynamics (QCD) was formulated [1, 2] and the phenomenon of asymptotic freedom [3, 4] was discovered, the applicability of this theory was limited by the physics of hard processes, in which the quark-gluon interactions at large momentum transfers (small characteristic distances) are described in terms of the QCD perturbation theory. At this early stage, QCD was able to predict only certain integral characteristics of hadronic physics, like the logarithmic evolution of the moments of parton distributions, or the total cross section of $e^+e^- \to hadrons$.

For the further development of QCD, understanding of nonperturbative nature of the quark-gluon vacuum was crucial. As a result, during last ten-fifteen years a remarkable progress was achieved in application of QCD to the physics of an individual hadron, i.e. to calculation of various hadronic characteristics: masses, coupling constants, form factors and structure functions. New methods of calculation, based directly on the QCD Lagrangian, have been suggested. Applying these methods, one deals with well defined objects, such as vacuum correlation functions of quark currents, and uses basic principles, such as analyticity and unitarity.

One of the most fruitful and universal approaches of direct calculation of hadronic parameters is the method of QCD sum rules [5]. Originally, this method was formulated to evaluate the mass of a given hadron H and its coupling to a corresponding quark current j_H, i.e. the matrix element $\langle H \mid j_H \mid 0\rangle$. The calculation starts from introducing the vacuum correlation function of two currents,

$$\Pi(q^2) = \int d^4x e^{iq\cdot x}\langle 0 \mid T\{j_H(x), j_H(0)\} \mid 0\rangle \ . \tag{1}$$

The main idea is to use two different representations for this correlator. Firstly, invoking the dispersion relation based on analyticity and unitarity, one is able to represent $\Pi(q^2)$ in a form of a sum

$$\Pi(q^2) = \frac{\langle 0 \mid j_H^+ \mid H\rangle\langle H \mid j_H \mid 0\rangle}{m_H^2 - q^2} + \int \frac{ds\ \rho_H(s)}{s - q^2}\ , \tag{2}$$

which starts from the lowest resonance contribution of the hadron H and includes dispersion integral over higher resonances and excited states with the quantum numbers of H, represented by the spectral density ρ_H. Secondly, using the operator product expansion (OPE) of the current product in (1) it is possible to calculate $\Pi(q^2)$ in a form of series

$$\Pi(q^2) = \sum_d C_d(q^2)\langle 0 \mid O_d \mid 0\rangle \tag{3}$$

of vacuum matrix elements of local, dimension d operators O_d composed from quark and gluon fields. The calculation is valid in the q^2 region of high virtuality, far enough from resonances and hadronic thresholds in the dispersion integral (2). The answer (3) includes, apart from the perturbative contribution of the unit operator with $d = 0$, nontrivial contributions of higher dimensional operators. The latter are interpreted as effects of interactions with nonperturbative quark-gluon fluctuations of the vacuum. The matrix elements $\langle 0 \mid O_d \mid 0\rangle$ represent average densities of vacuum fields, the so-called condensates.

Equating representation (2) with (3), one is able to express the measurable quantities, the mass of the hadron m_H and the coupling $\langle H \mid j_H \mid 0\rangle$ via QCD parameters: the quark masses, the quark-gluon coupling constant and the condensate densities. It is important that

the latter are universal and do not depend on the quark current j_H. For practical applications, one has to use certain technical improvements, so that the contributions of excited states to dispersion integral (2) and the contributions of higher-dimension condensates to the OPE (3) are simultaneously suppressed.

As a result of numerous applications of this method, the masses and couplings of many hadrons with various spin-parity and quark flavour content have been successfully reproduced. A few QCD parameters, the quark masses and the densities of the low dimension condensates serve as universal inputs in these calculations. They are determined from sum rules with experimentally known hadronic part and after that used in many other sum rules (for details see the original papers [5] and reviews [6, 7, 8]).

The main problem which is considered in this thesis is to generalize the method of QCD sum rules from calculating the characteristics of an individual hadron towards direct evaluation of the quantities measured in a given hadronic process. The characteristics of processes, such as decay amplitudes, form factors or cross sections, generically involve more complicated hadronic matrix elements, in many cases containing two or more different hadrons.

The way to obtain the sum rules for the simplest hadronic process, the radiative transition between charmonium levels was firstly suggested in [9]. The main idea of the method is to introduce a correlation function of three local currents, including two $\bar{c}c$ currents with appropriate quantum numbers, interpolating the charmonium levels, and an electromagnetic current of the photon emission. This correlator is an analytical function of two independent variables, the squared four-momenta of the quark currents, and has, therefore, a double dispersion relation. The double spectral density in the timelike region starts from the resonant contributions proportional to the amplitudes of the lowest radiative transitions between charmonium levels. One may then estimate the amplitudes after calculating the correlator in the spacelike region with the help of OPE. The major advantage of the obtained three-point sum rules is the following. They depend on the same set of universal parameters of QCD (quark masses, α_s, condensates) as the two-point sum rules outlined above. No new parameters should be introduced. In general, an interconnected hierarchy of QCD sum rules based on various two-, three- and four-current correlators emerges. It makes the predictive power of the method very impressive. In particular, transition to three-current correlation functions, allows one to calculate various form factors [10, 11] and three-point vertices. The further transition to the four-current correlators, provides a possibility to estimate the structure functions of hadrons [12] or two-photon cross-sections.

Various modifications of the QCD sum rule method suggested and developed in this thesis, are applied mainly to the processes with heavy hadrons. In many cases, the corresponding correlators with heavy quark currents are indeed better suited for OPE, due to intrinsic "storage of virtuality" provided by the large mass of c and b quarks. Some of the approaches, considered here, are nevertheless more general and may be useful also for processes with light hadrons. In addition, we consider hadronic processes with initial and final photons. The corresponding correlators can also be relatively easy analyzed in the framework of OPE, due to the pointlike nature of the photon.

The content of the thesis is presented in the following chapters, from 2 to 7.

In chapters 2 and 3 the processes with heavy quarkonium are considered. After formulating the double-dispersion-relation method for the amplitudes of radiative transitions we calculate the leading nonperturbartive effect of the gluon condensate and present our predictions for the lowest transitions in charmonium. An interesting analogy between QCD sum rules and radiative transitions in nonrelativistic quantum mechanics is pointed out. The obtained results for three-point correlation functions allow to estimate various annihilation processes of heavy quarkonium bound states, among them, the two-photon annihilation and the annihilation to the light scalar (Higgs-type) boson or axion. Finally, at the end of the chapter 3, we consider an alternative version of the sum rule, which uses the four-point correlation funciton and contains the two-photon widths of charmonium. This sum rule allows to estimate the gluon condensate density in a new independent way.

In chapter 4 we turn to investigation of the photon structure function, which is measured in two-photon processes. This chapter describes a new model-independent approach to the problem of evaluating the so called " hadronic part" of this structure function. The importance of measuring the structure function of the virtual photon in the region of intermediate values of the Bjorken variable x , is emphasized. The leading nonperturbative contribution to this structure function due to the gluon condensate is calculated. The structure function of the real photon is then obtained using dispersion relation in the virtual photon momentum. The result, without any new parameter, is in agreement with experimental data. In section 4.4, the part of the photon structure function, corresponding to the charm production is considered. In experimental analysis, this part is usually subtracted using the perturbative approximation. We demonstrate that from the point of view of QCD, the charm contribution is one of the most "pure"objects in the hadronic physics. Direct calculation of the moments of this structure function, presented here, reveals that the nonperturbative gluon condensate contribution influences these moments substantially, providing another interesting possibility to measure the condensate density.

In chapter 5 a new method of calculation of the heavy meson structure functions, i.e. of the valence heavy parton distribution in D or B mesons, is suggested. It is based on the 4-point correlation function and uses double dispersion relations. The resulting moments of parton distributions estimated from the obtained sum rules nicely agree with phenomenological expectations. The sum rules predict a distinct dependence of the parton distribution on the spin-parity of the heavy meson. The obtained results are then used to produce certain phenomenological output. Firstly, we estimate the fragmentation functions of heavy mesons and, secondly, comparing the obtained parton distributions with their parametrization in Regge-pole models, we predict the intercept of the heavy quarkonium trajectory.

Among most topical problems of hadronic physics, is the spin content of the proton. Experimantally, this problem is under intensive investigation in various processes with virtual polarized photons interacting with the nucleon target. The universality of the QCD sum rule method allows to attack this problem theoretically, in various ways. The first idea to study the structure functions of nucleon by using the OPE for four-current correlation function with baryon currents [13] was formulated in [12]. In chapter 6 we apply this technique to the direct calculation of the chirality violating structure function $h_1(x)$ of the nucleon in intermediate values of scaling variable x.

In sect. 6.3, we use another method, which allows to estimate static characteristics of hadrons, i.e. couplings with zero 4-momentum transfer, in the framework of OPE. One has to introduce an auxiliary external static field and to expand the product of currents in presence of this field [14, 15]. We study the sum rule for the nucleon $SU(3)$ flavour-singlet axial constant, directly connected with the fraction of proton spin carried by quarks. The result for this constant is obtained in terms of specific nonperturbative parameters, so called induced condensates. A problem of determining these parameters is investigated. An interesting, and physically important phenomenon of breaking of the OPE for the correlator of two singlet axial currents is indicated.

In chapter 7, as an alternative to the conventional QCD sum rules based on OPE in local operators, we use the expansion of the correlators around the light-cone. This expansion is extremely useful for exclusive hadronic processes containing a light meson (e.g. pion or kaon) or a real photon in the final state where the conventional method encounter principal difficulties in form of infinite series of local condensates. The QCD sum rules in this approach are obtained in terms of so-called light-cone wave functions [16]. The latter are certain universal matrix elements of nonlocal quark-gluon operators taken between the vacuum and the light hadron or photon state. The expansion goes over twists of the underlying quark-gluon operators. The light-cone wave functions play the role, which in conventional sum rules is played by the vacuum condensates. We start the applications of this method from the calculation of the heavy-to-light form factors $B \to \pi$ and $D \to \pi$ which are measurable in the semileptonic exclusive decays of heavy mesons. The form factors are obtained with twist 4 accuracy and for timelike momentum transfers up to the values of order of the heavy quark mass squared. Further application of the light-cone sum rule technique involves an estimate of the strong couplings of heavy mesons with pions, $D^*D\pi$ and $B^*B\pi$. Combinig the form factors calculated before at small and intermediate values of the momentum transfer with the pole-model approximation we predict the semileptonic widths $B \to \pi l\nu_l$ and $D \to \pi l\nu_l$ and extract the value of the fundamental CKM parameter V_{ub}. Finally, in the section 7.5 we demonstrate the use of the concept of the photon light-cone wave function. As a study case, we consider the long-distance contribution to the amplitude of the weak radiative decay $B \to \rho\gamma$. Physically, the effect corresponds to the photon emission from the initial B meson combined with the weak annihilation. The light-cone sum rule provides the first model-independent estimate of this long-distance effect. As a byproduct of this calculation, the widths of $B \to \mu\nu_\mu\gamma$ and $D \to \rho\gamma$ decays are predicted.

In the concluding chapter 8 the main results of the thesis are summarized. In the appendix, we list the publications, on which the thesis is based.

2 QCD sum rules for radiative transitions in charmonium

For several reasons, the radiative transitions between charmonium levels represent very convenient objects for application of QCD sum rules. These processes have simple initial and final states. The charmonium spectroscopy is well studied experimentally. The masses and the coupling constants of lowest charmonium states are calculable from two-point sum rules [5, 6, 7]. The corresponding correlation functions have well defined OPE and the most important nonperturbative effects are due to the interaction of c quarks with the gluon condensate.

2.1 The outline of the method

For definiteness we consider the radiative transitions between the pseudoscalar (η_c, η_c') and the vector ($J/\psi, \psi'$) levels of charmonium. Following [9], one introduces a three-point correlator:

$$\Delta_{\mu\nu}(p_1,p_2) = \int d^4x d^4y e^{-i(kx+p_2y)} \langle 0 \mid T\{j_1(0) j_\mu^{em}(x) j_{2\nu}(y)\} \mid 0\rangle$$
$$= \epsilon_{\mu\nu\alpha\beta} p_1^\alpha p_2^\beta \Delta(p_1^2,p_2^2) \ , \tag{4}$$

where $j_\mu^{em} = \bar{c}\gamma_\mu c$ is the electromagnetic current corresponding to the emission of a real photon with momentum k; $j_1 = \bar{c}i\gamma_5 c$ and $j_{2\nu} = \bar{c}\gamma_\nu c$ are the pseudoscalar and vector currents with momenta p_1 and p_2 ($p_1 = p_2 + k$, $k^2 = 0$). If one is interested in processes of transition between charmonium levels with other J^{PC} quantum numbers, the γ-matrix structure of the currents should be appropriately changed.

In the region $p_1^2, p_2^2 \ll 4m_c^2$ the invariant amplitude $\Delta(p_1^2,p_2^2)$ corresponds to a highly virtual fluctuation, developing at small distances of order of $1/(2m_c)$. In zeroth order in α_s, this amplitude is approximated by a three-point c–quark loop (Fig. 2.1a). In terms of the OPE this diagram corresponds to a Wilson coefficient of the unit operator in the expansion of the T-product of three currents in (4). The next important contribution to the OPE is the gluon condensate term, to be discussed below. Employing analyticity and unitarity for the invariant amplitude Δ one writes a double dispersion relation in two variables, p_1^2 and p_2^2:

$$\Delta(p_1^2,p_2^2) = \int ds_1 \int ds_2 \frac{\rho(s_1,s_2)}{(s_1-p_1^2)(s_2-p_2^2)} \ . \tag{5}$$

We neglect possible subtractions which are not important for the sum rule derivation. In the double spectral density,

$$\rho(s_1,s_2) = f_\psi f_{\eta_c} m_\psi A(J/\psi \to \eta_c\gamma)\delta(s_1 - m_{\eta_c}^2)\delta(s_2 - m_\psi^2)$$
$$+\{J/\psi \to \psi', \eta_c \to \eta_c'\} + \rho^h(s_1,s_2) \ , \tag{6}$$

we single out the lowest pole in the variable p_1^2 corresponding to the lightest pseudoscalar ($J^{PC} = 0^{-+}$) charmonium level, η_c, and the lowest pole in the variable p_2^2 corresponding to the lightest vector ($J^{PC} = 1^{--}$) level, J/ψ. The residue of this double pole contribution is proportional to the product of the decay constants of J/ψ and η_c

$$\langle 0 \mid \bar{c}i\gamma_5 c \mid \eta_c\rangle = m_{\eta_c} f_{\eta_c} \ ,$$
$$\langle J/\psi \mid \bar{c}\gamma_\nu c \mid 0\rangle = m_\psi f_\psi \epsilon_\nu^\psi \ , \tag{7}$$

and the radiative transition amplitude, the main object of our interest, defined as

$$\langle J/\psi(p_2) \mid j_\mu^{em} \mid \eta_c(p_1)\rangle = m_{\eta_c}^{-1} \varepsilon_{\mu\nu\alpha\beta} \epsilon_\psi^\nu p_1^\alpha p_2^\beta A(J/\psi \to \eta_c\gamma) \ , \tag{8}$$

where ϵ^ψ is the polarization vector of J/ψ. The analogous contributions of radiative transitions between the excited states $\psi'(3685)$, $\eta_c'(3590)$ and the lowest states are denoted with parenthesis. Diagrammatical representation of this part of the spectral function is presented in Fig. 2.2. The spectral density $\rho^h(s_1,s_2)$ includes amplitudes of transitions from the higher states to the lowest states, and of transitions between the higher states. In both channels, we attribute to the set of higher states the charmonium levels located above the open charm threshold $\sqrt{s_0} = 2m_D$, as well as the continuum of two- and many-body states of charmed -anticharmed mesons with J^{PC} quantum numbers of the respective channels. Standard approximation [5] following from the quark-hadron duality, is to replace the spectral density ρ_h by the corresponding density of the lowest order perturbative (partonic) contribution to OPE. For the correlator considered here, one has to replace

$$\rho^h(s_1,s_2) \simeq \rho^0(s_1,s_2)\Theta(s_1-s_0)\Theta(s_2-s_0) \tag{9}$$

where ρ^0 is the double spectral density of the three-point diagram of Fig. 2.1a, corresponding to the lowest order perturbative contribution to OPE of the correlator (4). After substituting (6) and (9) into the r.h.s. of (5), in order to suppress the duality-dependent contributions of higher states and to get rid of possible subtraction terms, we use multiple differentiation of the dispersion relation in both variables, p_1^2 and p_2^2. In other applications, one may also use the Borel transformation [5]. After differentiation at $p_1^2 = p_2^2 = 0$ which is still far enough from the physical thresholds $p_1^2, p_2^2 \sim 4m_c^2$ one finally obtains the power moments of the QCD sum rule for the linear combination of four transition amplitudes:

$$A(J/\psi \to \eta_c\gamma) + \frac{f_{\eta_c'}}{f_{\eta_c}}\left(\frac{m_{\eta_c}}{m_{\eta_c'}}\right)^{2n+2} A(\eta_c' \to J/\psi\gamma)$$
$$+\frac{f_{\psi'}}{f_\psi}\left(\frac{m_{J/\psi}}{m_{\psi'}}\right)^{2k+1}\left[A(\psi' \to \eta_c\gamma) + \frac{f_{\eta_c'}}{f_{\eta_c}}\left(\frac{m_{\eta_c}}{m_{\eta_c'}}\right)^{2n+2} A(\psi' \to \eta_c'\gamma)\right]$$
$$= \frac{3m_\psi m_{\eta_c}}{2\pi^2 f_\psi f_{\eta_c}}\left(\frac{m_\psi}{2m_c}\right)^{2k}\left(\frac{m_{\eta_c}}{2m_c}\right)^{2n+1}\frac{[(n+k)!]^2}{(2n+2k+2)!}\left\{1 + O(\alpha_s) + c_{nk}^G + ...\right\} \tag{10}$$

where $O(\alpha_s)$ denotes unaccounted hard gluon corrections to the unit operator (Fig.2.1b), c_{nk}^G is the gluon condensate contribution, ellipses in the l.h.s. correspond to the integral over ρ^h and ellipses in the r.h.s. denote higher dimension contributions to the OPE. The presented derivation illustrates only the basic ingredients of the method. Further details can be found in [9, 17].

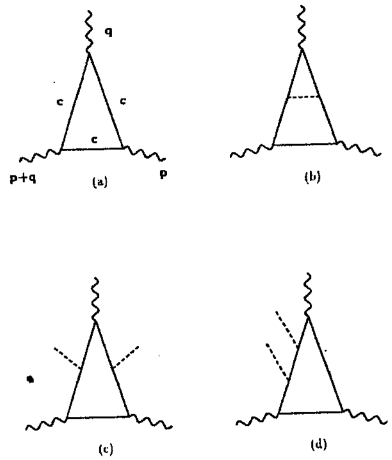

Fig. 2.1 *(a) The diagram corresponding to the unit operator contribution of the OPE of the correlator (4), (b) - one of the diagrams of $O(\alpha_s)$ corrections, (c) the diagrams corresponding to the gluon condensate contribution.*

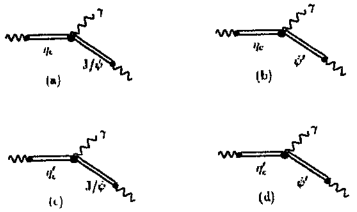

Fig. 2.2 *The diagrammatic representation of the double spectral function (6).*

2.2 The gluon condensate contribution

The second important contribution to the OPE of the correlation function (4) after the unit operator, is due to the gluonic operator $G^a_{\mu\nu}G^{a\mu\nu}$ with dimension 4. Physically, this contribution takes into account the interactions of c quarks with the vacuum gluon condensate. Due to the dimension of the operator, this effect is suppressed by the fourth power of the heavy quark mass (the characteristic scale of the correlation function). It is therefore inessential for heavier b quarkonium. Besides that, in the b quarkonium the role of higher order perturbative corrections of the Coulomb type is enhanced. Therefore, charmonium is a unique optimal

object for probing the gluon condensate. This circumstance was used in [5] to extract the value of the gluon condensate density

$$\langle 0 \mid \frac{\alpha_s}{\pi} G^a_{\mu\nu}G^{a\mu\nu} \mid 0\rangle \simeq 0.012 \text{ GeV}^4 , \tag{11}$$

from the sum rules for the product of two $\bar{c}\gamma_\nu c$ currents saturated by the experimentally measured contributions of $J/\psi, \psi', \ldots$ resonances.

In order to obtain the gluon condensate contribution to (10), one needs to calculate various quark loop diagrams in an external gauge field. For this kind of problems a useful tool is the Fock-Schwinger gauge [19, 20] for the gluon condensate field A^a_μ used in [21, 22, 23, 24]:

$$(x - x_0)_\mu A^{a\mu} = 0 \tag{12}$$

Here we fix the point $x_0 = 0$. The fixed point gauge allows to express the field four-vector directly in terms of the field strength tensor,

$$A^a_\mu(x) = x^\tau \int_0^1 udu G^a_{\tau\mu}(ux) = \frac{1}{2}x^\tau G^a_{\tau\mu}(0) + \frac{1}{3}x^\sigma x^\tau \mathcal{D}_\sigma G^a_{\tau\mu}(0) + \ldots, \tag{13}$$

at the expense of violating the translational invariance. The latter is naturally restored in the sum of all diagrams which should be gauge-independent. For the calculation discussed here, the first term in the expansion (13) is sufficient. The Wilson coefficient of the gluon condensate contribution to the three-point correlator (4) corresponds to six diagrams, some of them shown in Fig. 2.1c,d. The fixed point gauge (12) allows already at the initial stage to represent the answer for the diagrams in a form of vacuum average of two gluon field tensors multiplied by a standard Feynman integrals. For brevity, we do not present here the complete answer for the diagrams in a form of double integral over parameters obtained in [17]. For the correlator interpolating $0^{++} \to 1^{--}$ transitions, the corresponding results can be found in [18]. For the three-current correlator interpolating the decay of $\chi_{c2} \to 2\gamma$ (the case of 2^{++} current instead of the 0^{-+} current in the correlator (4) at $p_2^2 = 0$) the gluon condensate contribution is calculated in [25].

After differentiation in $p^2_{1,2}$ at $p_1^2 = p_2^2 = 0$ the final result for the gluon condensate contribution to the sum rule (10) is :

$$c^G_{n,k} = -\frac{(n+1)}{(2n+2k+1)}\left[(n+k)^3 - 2n^3 + 2k^2 + 4nk + 3n - k - 2\right]\Phi . \tag{14}$$

where the dimensionless parameter

$$\Phi = \frac{4}{9}\frac{\langle 0 \mid \frac{\alpha_s}{\pi} G^a_{\mu\nu}G^{a\mu\nu} \mid 0\rangle}{(4m_c^2)^2} \tag{15}$$

One easily notices that the gluon condensate contribution fastly grows as $(n+k)^3$ and is negative with respect to the loop diagram answer. The analogous contribution for the correlator with the scalar current has the same behaviour, and turns out to be numerically larger. The indicated behaviour is in accordance with the result of [5, 7] obtained for the two-point correlators with pseudoscalar,vector and scalar $\bar{c}c$ currents. The enhancement of the c-quark interactions with the gluon condensate in the case of the scalar current has the following physical interpretation: being in P wave, the quarks are propagating at larger average distances.

The perturbative $O(\alpha_s)$ corrections to the coefficient of the unit operator have been calculated [26] only for selected moments . One has to evaluate two-loop three-point diagrams with massive quarks, as shown in Fig. 2.1b . The complete calculation is very important for further improvement of sum rules. It may be recommended as a useful application of recently developed technology of two-loop calculations using various computer-algebra devices. Meanwhile, from the existing results for two-point correlators [5, 7] and from estimates of [26] one may conclude that the magnitude of perturbative corrections to the sum rule (10) is of order of 10-15%. Therefore, the sum rules presented here predict radiative decay widths with a typical accuracy about 20-30 %.

2.3 Estimates of radiative transition widths

One easily recognizes that at sufficiently large n and k (in fact, the combination $n+k$ plays the role of effective number of the moment) the contributions of excited and higher states to the sum rule (10) are suppressed by powers of ratios like $m_\psi/m_{\psi'}$. Therefore one may safely use experimental data and/or phenomenological assumptions, like quark-hadron duality, to estimate these contributions. At $(n,k)=(3,3),(3,4),(4,3)$ one expects [17] that all contributions to the l.h.s. of (10) sum up to a $O(5\%)$ correction to the $J/\psi\to\eta_c\gamma$ amplitude. Neglecting this small "contamination" one has then from the r.h.s. of (10) the following estimate

$$A(J/\psi\to\eta_c\gamma)=3.3\div 3.5 \tag{16}$$

For the numerical analysis we have selected the moments with $n+k\le 7$ for which the gluon condensate contribution is still less than 30%. We have also used the values of the c quark mass and coupling constants f_ψ and f_{η_c} obtained from the two-point sum rules [5]. Therefore, the sum rule (10) does not contain new parameters. In the adopted normalization, the estimate (16) yields

$$\Gamma(J/\psi\to\eta_c\gamma)=1.8\div 2.0\ \mathrm{keV} \tag{17}$$

Independent analysis of the sum rule (10) carried out in [26] have confirmed our result within 20-30 %. A very close estimate was obtained in [27] where the vector dominance type relation was used, inspired by the sum rules. Recent analysis in the relativistic quark model [28] predicts a width close to (17). We remind that the sum rule method is purely relativistic and does not refer to any picture of charmonium potential and/or wave functions.

The single, already quite old experimental measurement [29] still used in [30] gives $BR(J/\psi\to\eta_c\gamma)=1.3\pm0.4\%$. Combined with the current value of the total J/ψ width [30] one has $\Gamma(J/\psi\to\eta_c\gamma)=1.1\pm0.3$ keV. Taking into account agreement among various theoretical calculations (see e.g. review [31]) one concludes that a new measurement of the $J/\psi\to\eta_c\gamma$ is of a great demand.

In analogous way, the sum rules for radiative transitions between scalar (χ_{c0} with J^{PC}) and vector ($J/\psi,\psi'$) levels of charmonium have been obtained [9, 18]. Note that in the nonrelativistic potential models these processes represent electric-dipole-type transitions sensitive to the overlap of wave functions, whereas the pseudoscalar-vector transitions considered above, are of the magnetic-dipole type. From the point of view of sum rules the basic difference is in the initial currents in the correlator (4). One has to replace the pseudoscalar current by the scalar current $\bar{c}c$. The final form of the moments of sum rules at the point p_1^2 $p_2^2=0$ is

$$A(\chi_{c0}\to J/\psi\gamma)+\frac{f_{\psi'}}{f_\psi}\left(\frac{m_\psi}{m_{\psi'}}\right)^{2k+1}A(\psi'\to\chi_{c0}\gamma)+\ldots=\frac{3}{\pi^4}\frac{m_\psi^{4k+1}m_{\chi_{c0}}^{2n+4}}{f_\psi f_{\chi_{c0}}m_c^{4n+4k+1}}\frac{(n+1)!(n+k)!}{(2n+2k+4)!}$$

$$\times\left[(2k+3)(n+k+1)+1\right]\left\{1+O(\alpha_s)+d^G_{n,k}+\ldots\right\} \tag{18}$$

where the expression for the gluon condensate term $d^G_{n,k}$ can be found in [18].

The practical application of this sum rule turns out to be more difficult because there are no moments having the contribution of higher states and, simultaneously, the gluon condensate contribution sufficiently suppressed. One has to follow [7] and expand the dispersion relations at negative values of $p^2_{1,2}=-4m_c^2$. As numerical analysis reveals [18], it is possible to single out six optimal moments in the interval $8\le(n+k)\le 10$. The obtained linear system of equations for two amplitudes is almost degenerate and fits the following relation

$$A(\chi_{c0}\to J/\psi\gamma)+0.24\ A(\psi'\to\chi_{c0}\gamma)=5.1 \tag{19}$$

The resulting prediction for amplitude of the lowest transition $A(\chi_{c0}\to J/\psi\gamma)$ noticeably depends on the second amplitude $A(\psi'\to\chi_{c0}\gamma)$. If one uses the absolute value of the latter extracted from the experimentally measured width [30] the result for the lowest transition has two solutions dependent on the relative sign between two amplitudes in (19). The solution with the negative sign yields $\Gamma(\chi_{c0}\to J/\psi\gamma)\simeq 130$ keV which is in a reasonable agreement with experimentally measured width $\Gamma(\chi_{c0}\to J/\psi\gamma)=92\pm25$ keV [30]. The solution corresponding to the positive sign is certainly rejected by the data. The accuracy will be improved when the perturbative corrections are calculated.

2.4 "Asymptotic freedom" for radiative transitions in quantum mechanics

A very useful analogy between QCD and nonrelativistic quantum mechanics, was noticed in [32]. If the interaction potential is nonsingular at small distances and grows sufficiently fast at $r\to\infty$, one is able to derive asymptotic sum rules in quantum mechanics, which are, on one side saturated by lowest energy levels , and, on the other side, well reproduced by Born approximation of the potential. In [33] it was shown that the abovementioned analogy can be generalized also to the radiative transitions. One is able to derive double sum rules in quantum mechanics which are, on one side, saturated by the contributions of the lowest electrical-dipole transitions, and on the other side, well approximated by Born terms taken over the interaction potential. The starting object for derivation is a correlation function of two nonrelativistic temporal Green functions and the velocity operator. The latter corresponds to dipole radiation of the nonrelativistic particle. Using the spectral representation of the Green function, one is able to derive an expression for this correlation function in the form resembling nonrelativistic limit of the l.h.s (the hadronic part) of the sum rules (18). The contribution of each dipole $P\to S$ transition is proportional to the corresponding overlap integral I_{PS}, (i.e. to the amplitude of the radiative transition in the nonrelativistic approximation), multiplied by the S level wave function $R_S(r)$ and derivative of the P level wave function $R'_P(r)$ at the origin $r=0$. Simulteneously, one is able to evaluate this correlation function in a form of " operator expansion" series in the interaction potential. For this purpose one uses the corresponding expansions for the Green functions. As an example we present the obtained [33] sum rule for the oscillator potential $V=m\omega^2r^2/2$:

$$\frac{1}{2\pi}R'_{1P}(0)R_{1S}(0)I^{11}_{PS}sh\left(\frac{E_{1P}-E_{1S}}{2\epsilon}\right)\exp\left(-\frac{E_{1P}+E_{1S}}{2\epsilon}\right)+\ldots$$
$$=\left(\frac{m\epsilon}{2\pi}\right)^{3/2}\left[1-\frac{\omega^2}{4\epsilon^2}+\frac{19\omega^4}{480\epsilon^4}+\ldots\right] \tag{20}$$

where $2\epsilon = \epsilon_1 = \epsilon_2$, $\epsilon_{1,2}$ being the Borel transformation parameters in S and P channels, respectively. This transformation is used to suppress the higher state contributions. Effectively, ϵ_1 (ϵ_2) correspond to $4m_c^2/k$ ($4m_c^2/n$) in the QCD sum rules presented above. In the r.h.s. of (20) the first three terms of Born expansion in the oscillator potential are retained. The fact that the sum rule depends on the reduced parameter ϵ is in full analogy with the effective $(n+k)$ dependence of the QCD sum rules for radiative transitions. Both parts of the nonrelativistic sum rule (20) agree with each other in the region $0.6 < \epsilon/\omega < 1.0$ if one retains in the "physical", l.h.s. the three lowest dipole transitions and uses the values of the overlap integrals and wave functions obtained from solving the Schrödinger equation for the oscillator potential.

Summarizing, the QCD sum rules for the radiative transitions based on OPE of the three-current correlator, and on double dispersion relations, have a definite prototype in nonrelativistic quantum mechanics.

3 Photonic annihilation of heavy quarkonuim

3.1 Two-photon decay amplitudes from three-point QCD sum rules

The process of annihilation of O^{-+} charmonium level η_c into two photons can be evaluated in QCD starting from the three-point correlator (4) considered above, in the particular case $p_2^2 = 0$, i.e. when the correlator corresponds to the interaction of the heavy quark-antiquark current with two real photons. The same is possible for photonic annihilation of O^{++} or 2^{++} charmonium states, χ_{c0} or χ_{c2}, if one replaces the pseudoscalar $\bar{c}c$ current by a scalar or tensor one, respectively. Naturally, in this derivation the dispersion relation in one variable p_1^2 is sufficient. This idea was suggested in [6] where the three-point sum rules without gluon corrections have been obtained. The account of the gluon condensate contribution substantially improves the sum rules. For the $\eta_c \to 2\gamma$ decay the necessary calculation was done in [34, 35]. From the correlator with the scalar current we obtained [18] the sum rule for the $\chi_{c0} \to 2\gamma$ decay:

$$A(\chi_{c0} \to 2\gamma) + ... = \frac{3m_{\chi_{c0}}}{\pi^2 f_{\chi_c}} \left(\frac{m_{\chi_{c0}}}{m_c}\right)^{2n+1} \frac{(n!)^2}{(2n+4)!}(3n+4)(1+d_{n0}^G\phi+...) : \qquad (21)$$

where d_{n0}^G is the same gluon condensate correction as in (18). For brevity, we present here the compact expression obtained after taking moments at $p_1^2 = 0$. As in the case of radiative transition considered above, in order to avoid too fastly growing gluon condensate contribution, one has to expand this sum rule in moments in the spacelike point $p_1^2 = -4m_c^2$. This yields:

$$\Gamma(\chi_{c0} \to 2\gamma) = 3.0 \pm 0.4 \text{ keV} . \qquad (22)$$

The indicated uncertainty corresponds to the spread over moments. The obtained prediction is close to the independent estimate presented in [7]. The current experimental value [30] $\Gamma(\chi_{c0} \to 2\gamma) = 5.6 \pm 3.8$ keV still has a large uncertainty. We stress that in the absence of perturbative $O(\alpha_s)$ correction to (21) the prediction (22) has additional uncertainty of order of 20-30 % . Within this uncertainty and within experimental error, the agreement between sum rule prediction and the experimental number is quite satisfactory taking into account that no new parameters are involved. It would be very interesting to measure the width of the $\chi_{c0} \to e^+e^-\gamma$ " Dalitz decay" for which the sum rules [18] predict a ratio

$$\frac{\Gamma(\chi_{c0} \to e^+e^-\gamma; m_{e^+e^-} < 1\text{GeV})}{\Gamma(\chi_{c0} \to 2\gamma)} \simeq \frac{4\alpha}{3\pi} I , \qquad (23)$$

where the numerical value of the phase-space integral is $I = 5.3$.

Furthermore, in [25] the sum rules for two independent invariant amplitudes of the $\chi_{c2} \to 2\gamma$ decay were derived taking into account the gluon condensate contribution. A new diagram should be added in this case to Fig. 2.1c , corresponding to the soft gluon emission directly from the 2^{++} vertex. The origin of this diagram is due to covariant derivative in the current with tensor quantum numbers. The estimate of the width obtained for this decay is

$$\Gamma(\chi_{c2} \to 2\gamma) = 2.35 \pm 0.2 \text{ keV} . \qquad (24)$$

The experimental number for this width has recently dropped down to the interval between 0.3 [30, 36] and 0.7 keV [37] which is considerably lower than (24). Since our prediction is obtained without fitting any parameters, specific for this channel, the origin of the discrepancy, (if the current data will be confirmed) may be a reflection of the fact that for the correlator with 2^{++} current the role of higher order OPE terms and perturbative corrections is considerably higher than expected. Clearly, this interesting problem deserves further investigation.

In conclusion of this subsection, we consider the electromagnetic annihilation widths in b -quarkonium. Although the method developed above is in general not applicable to b quarkonium, one may use the similarity of vector and pseudoscalar levels, Υ and η_b. Taking into account the most important , Coulomb part of the perturbative α_s-correction, we obtained [25] the following estimate:

$$\Gamma(\eta_b \to 2\gamma) = (1.01 \pm 0.1) 3 Q_b^2 \Gamma(\Upsilon \to \mu^+\mu^-) \simeq 0.4 \text{ keV} \tag{25}$$

from the ratios of first few moments of the corresponding three- and two-point sum rules. As anticipated, the result turns out to be very close to the nonrelativistic limit which corresponds to numerical coefficient in parenthesis in (25) equal to 1.

3.2 Heavy quarkonium decay to a light scalar boson or axion

In various models with spontaneous symmetry breaking new light pseudoscalar (P) bosons (axions) or light scalar (S) bosons are predicted. In many of these models the coupling of these particles to quarks is proportional to the quark mass, similar to the Higgs coupling in the standard electroweak theory. Therefore, one of the best methods to detect such particles is to search for corresponding radiative decays of quarkonia, e.g. $J/\psi \to P\gamma, S\gamma$, or $\Upsilon \to P\gamma, S\gamma$. A simple formula for the width of such decays was suggested in [38]

$$\Gamma_W(V \to P\gamma) = \Gamma_W(V \to S\gamma) = \frac{m_Q^2}{2\pi\alpha x_Q^2} \Gamma(V \to \mu^+\mu^-) \equiv \Gamma_W(V) , \tag{26}$$

where $V = J/\psi, \Upsilon (Q = c, b)$, x_Q is an electroweak model-dependent factor. The above formula was obtained in the approximation of nonrelativistic quarkonium model. In [39] the widths of quarkonium decays to photon and light scalar boson or axion were calculated by means of QCD sum rules. The idea of calculation is the following. In case of the pseudoscalar boson one starts directly from the three-point correlation function (4) with the squared momentum in the pseudoscalar channel $p_1^2 = 0$. For simplicity, we consider massless bosons. If $m_{S,P} \ll m_Q$, the same derivation is valid at $p_1^2 = m_{S,P}^2$.

After that, the sum rule in the vector channel is considered. For the scalar boson one has only to replace the pseudoscalar current by the scalar one in the same correlator. Having at our disposal the relevant OPE results for the correlators, and a sufficient amount of experimental information about resonances in the vector channels of charmonium and b quarkoniuum, we analysed the sum rules. For higher accuracy, we included the Coulomb part of the perturbative α_s-correction in the sum rules. As a result, the following numerical estimates for the decay widths in units of the width (26)were obtained

$$\Gamma(J/\psi \to P\gamma) = (1.20 \div 1.30) \Gamma_W(J/\psi) , \tag{27}$$
$$\Gamma(\Upsilon \to P\gamma) = (1.05 \div 1.15) \Gamma_W(\Upsilon) , \tag{28}$$
$$\Gamma(J/\psi \to S\gamma) = (0.75 \div 0.80) \Gamma_W(J/\psi) , \tag{29}$$
$$\Gamma(\Upsilon \to S\gamma) = (0.85 \div 0.90) \Gamma_W(\Upsilon) . \tag{30}$$

These model-independent estimates reveal that corrections to the nonrelativistic approximation (26) are noticeable, especially for charmonium decays.

3.3 Two-photon charmonium widths from four-point correlators

The four-point correlator is, by its complexity, the next after the three-point correlator in the QCD sum rule hierarchy. The imaginary part of the four-point correlator in the case of forward scattering is proportional to the total current-current cross-section.

Here we consider the simplest correlator of that kind, corresponding to the two-photon charm production cross-section:

$$D_{\mu\nu\lambda\rho} = -i(eQ_c)^4 \int d^4x d^4y d^4z \, e^{ip(x-y)-iqz}$$
$$\times \langle 0 \mid T\{j_\rho(0) j_\mu(x) j_\nu(y) j_\lambda(z)\} \mid 0 \rangle , \tag{31}$$

where $j_\rho = \bar{c}\gamma_\rho c$, $p^2 = q^2 = 0$, the kinematics corresponds to forward scattering of currents ($t = 0$) at fixed $s = (p+q)^2$. Q_c is the c quark electric charge.

The hadronic representation of the correlator (31) averaged over polarizations is simply a dispersion integral of the experimentally measurable cross-section $\sigma_{\gamma\gamma} = \sigma_{tot}(\gamma\gamma \to charm)$. The resonance contributions to this cross-section are received from all C -even charmonium levels which are able to annihilate into two photons. The calculation of the correlation function (31) in the spacelike region $-\infty < s \ll 4m_c^2$ provides an independent possibility to estimate the two-photon widths of charmonium. This possibility was used in [6] in zeroth approximation on α_s. In [40] we calculated the gluon condensate contribution to this sum rule, that is, the amplitude of the photon-photon scattering via intermediate c-quarks in the external vacuum gluon field. The details of the calculation will be presented in the next section for more general case. Here we present only the final form of the sum rule for two-photon widths:

$$M^n(s) = 8\pi^2 \sum_R \frac{(2J+1) m_R \Gamma(R \to 2\gamma)}{(m_R^2 - s)^{n+1}} + \int_{s_0}^{\infty} \frac{d\tilde{s}}{(\tilde{s} - s)^{n+1}} \tilde{s} \sigma^h_{\gamma\gamma}(\tilde{s}) . \tag{32}$$

In the sum over resonances R, the lowest contributions are provided by $\eta_c(2980)$, $\chi_{c0}(3415)$, $\chi_{c2}(3550)$, and $\eta_c'(3594)$ located below the open charm threshold. The cross-section $\sigma^h_{\gamma\gamma}(s)$ corresponds to open charm production in two photon collisions. In duality approximation, it can be replaced by the corresponding partonic cross-section. At large n the accuracy of the latter approximation is not very important. The expression for the l.h.s. of (32) at $s = 0$ is :

$$M_n(0) = 24\pi\alpha^2 Q_c^4 \{M_n^0 - M_n^{\langle G^2 \rangle} \Phi\} , \tag{33}$$

where the perturbative loop contribution is

$$M_n^0 = 2^{2n} \frac{(n!)^2}{(2n+3)!(2m_c)^{2n}} \left\{ 2\frac{n^2+5n+5}{n+2} + \frac{(5n+3)(n+1)}{n^2} \right\} . \tag{34}$$

The Wilson coefficient at the gluon condensate contribution is determined by

$$M_n^{\langle G^2 \rangle} = 2^{2n} \frac{[(n+1)!]^2}{(2n+3)!(2m_c)^{2n}}$$
$$\times \left\{ \frac{1081}{33} + \frac{16}{3} \left[\frac{5}{n+1} + \frac{1}{n+4} \right] + \frac{(4n^2+36n-45)(2n+1)(2n+3)}{16n(n+1)} \right\} , \tag{35}$$

where Φ is defined in (15). It is easy to convince oneself that at large n the ratio of the gluon condensate contribution to the unit-operator one grows approximately proportional to n^3, as in the case of the two-point correlator considered in [5] or in the case of three-point correlators considered above.

3.4 New possibility to determine the gluon condensate density

In [5] the numerical value of the gluon condensate density was extracted from the sum rules for the correlation functions of two e.m. c quark currents. The sum rules (32) obtained in [40] provide a new, independent possibility to estimate this key parameter of nonperturbative QCD. One has to substitute the necessary experimental data to the r.h.s. of (32) and compare it with the l.h.s. calculated in QCD according to (33). In practice, as it was noticed in [40] it is more convenient to use the ratios of neighbouring moments of (32) $r_n = M_{n+1}/M_n$ in order to get rid of large powers of m_c , the parameter which is known with limited accuracy. Comparing r_n calculated from (32) and (33) one may fit the gluon condensate density.

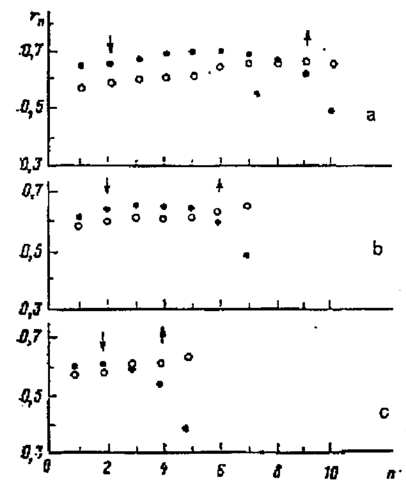

Fig. 3.1 *Comparison of the ratio r_n obtained in two ways: from l.h.s. (black circles) and from the r.h.s (white circles) of the sum rule (32) for various n. The arrows restrict the region of reliability of the sum rules. Part (b) corresponds to the standard choice (11) of the gluon condensate , and (a) and (c) correspond to values which are half and twice as large, respectively.*

Currently, the experimental values of two-photon charmonium widths are still very uncertain and have a typical error between 30% and 100%. Also the OPE result for of M_n still has room for improvement. One may expect that the total uncertainty of the theoretical part of the sum rule (32) is of order of $10 \div 15\%$, which is mainly due to lack of perturbative α_s corrections. Therefore, the full-scale task of extraction of the gluon condensate density should be postponed until one calculates the perturbative corrections and sufficiently precise measurements of the widths are available. At present stage, the sum rules (32) were used [40] for an important check of the consistency of the approach. Applying three-point sum rules [25, 34, 35] for each of two-photon annihilation amplitudes we fixed the widths entering the hadronic part of (32) theoretically, within accuracy of the three-point sum rules. After these substitutions and taking $\sqrt{s_0} = 3.8$GeV we compared the r.h.s. and the l.h.s. of the sum rules

(32). The results in form of ratios r_n are presented in Fig. 3.1 for different values of gluon condensate. It is evident that there is an agreement at the expected level of 10% for all those moments for which the approximation we have used is reasonable i.e. the higher states and condensate have simultaneously small contributions.

Importantly, the best agreement is achieved for the value of the gluon condensate (11) estimated in [5] and considered as a standard in all current sum rule calculations. Changing this value by factor of two in both directions, one increases the deviation between two sides of the sum rule (32) beyond expected accuracy of $10 \div 15\%$. This check, in our opinion, demonstrates very nontrivial self-consistency of the method.

Summarizing , we conclude that QCD sum rules for charm production in two-photon collisions provide new, diverse possibilities to analyze the properties of QCD vacuum. This analysis may be done in two ways: first, by comparing the calculated correlators with experimental cross-sections, and second by comparing various sum rules for the same two-photon widths.

4 Photon Structure Function in QCD

The photon structure function determines hadroproduction cross-section in the scattering of a virtual photon on a real photon. As in the case of e^+e^- annihilation, the physics is simplified due to absence of initial hadrons. In order to sum over final hadrons, one may use the parton model reducing the result, in first approximation, to a simple process $\gamma\gamma^* \to \bar{q}q$. Nevertheless, it is well known that apart from partonic part, the photon structure function contains also so called hadronic part, which cannot be calculated by means of QCD perturbation theory. This part is usually estimated with the help of the vector dominance model. The OPE method provides a new possibility [41, 42] to calculate the photon structure function. The result contains, apart from usual perturbative part, important nonperturbative contributions, which are dual to the hadronic part.

4.1 Structure function of the virtual photon

The cross-section of hadroproduction in two-photon collision is given by the imaginary part of the forward scattering amplitude. The expression for the latter is already given above in (31) in a form of correlator of four e.m. quark currents. One only needs to replace the c quark currents by the sum over light-quark currents $j_\rho = \sum_q \bar{q}\gamma_\rho q$ $(q = u, d, s)$ and slightly change the kinematical conditions. Hereafter we assume the light quarks to be massless. Consider first the scattering of two virtual photons at $q^2 \gg p^2$, $q^2, p^2 < 0$ and simultaneously, $|p^2| \gg \Lambda^2$. As it was shown in [13, 43], the method of OPE may be directly applied to the imaginary part of the correlator in the variable $s = (p+q)^2$, in the region of intermediate values of Bjorken variable $x = -q^2/2(p\cdot q)$ not too close to $x = 1$ and to $x = 0$. Apart from the unit operator, with a short-distance coefficient corresponding to the simple four-point quark loop (Fig. 4.1a,b), one encounters in this expansion the operator of gluon condensate. The short-distance coefficient for the latter corresponds to the sum of diagrams similar to ones shown in Fig. 4.1c,d. The contributions of quark- , quark-gluon and 4-quark condensates in leading $O(\alpha_s)$ are proportional to $\delta(1-x)$ and are not essential at intermediate x. The calculation of the gluon condensate contribution is usually done in the Fock-Schwinger gauge described above. It differs from the analogous calculation for the two-point or three-point correlators by some technical complications. The imaginary part in s of the diagrams similar to Fig. 4.1c,d is taken with the help of Cutkosky rules. The quark lines with vacuum gluon insertions and, respectively, with double poles in the propagators, have imaginary parts given by derivatives of δ-functions. The result of the calculation [41] expressed in terms of the structure function of the transverse virtual photon, yields following relation which is applicable at intermediate values of x:

$$F_2^T(x,p^2) = \frac{3\alpha}{\pi}\sum_q (e_q)^4 x \Big\{ -2 + 8x(1-x) + \left[x^2 + (1-x)^2\right] ln\frac{Q^2}{-x^2p^2} - \frac{4\pi^2}{27}\frac{\langle 0 \mid \frac{\alpha_s}{\pi} G^a_{\mu\nu}G^{a\mu\nu} \mid 0\rangle}{p^4x^2} \Big\} \qquad (36)$$

where the contribution of the gluon condensate is taken in the leading $\sim 1/p^2$ twist approximation. Note that the quantity $F_2^T(x,p^2)$ is in principle measurable by itself, demanding however great experimental efforts. One has to detect two tagged electrons in the final state of the two-photon process and to measure a cross section which is suppressed by virtualities of photons. This is a task for future colliders with upgraded luminosity. One has to stress that

the structure function of the virtual photon at intermediate x predicted in (36) is a unique object that can be simultaneously measured and calculated in QCD with sufficient accuracy. Another interesting object, the structure function of the longitudinal virtual photon, is studied in [41] by the same method.

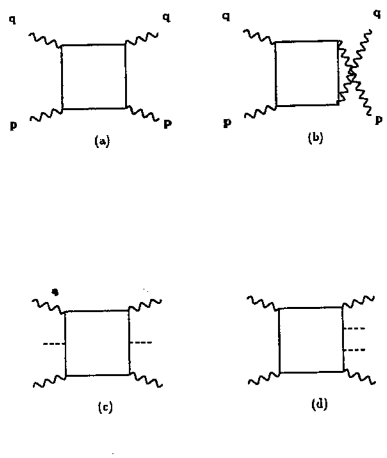

Fig. 4.1 *(a),(b) the diagrams determining the perturbative part of the photon structure function (coefficient of unit operator in OPE); (c),(d) examples of diagrams corresponding to the gluon condensate contribution*

4.2 Hadronic part of the photon structure function

In order to obtain the structure function of the real photon, the following method was suggested [42]. Using dispersion relation in p^2 one has to express the virtual photon structure function in terms of the integral over hadronic states. In this representation a standard model of the hadronic spectrum is used (in $SU(3)$ flavour approximation) : the lowest vector resonance with mass $m_V = m_\rho$ plus continuum of hadronic states with ρ quantum numbers. The latter is estimated with the help of quark-hadron duality, from the spectral density of the loop diagram. The model describes reasonably well the hadronic spectrum in the two-point sum rule for two e.m. light quark currents with isospin 1 [5]. From the latter sum rule, the value of the threshold parameter $s_0 = p_0^2 = 1.5$ GeV^2 is known. The other parameters of the dispersion representation are fixed by the OPE answer (36) at sufficiently large p^2. After that, making use of the analyticity of the dispersion relation in p^2, one may continue it to the point $p^2 = 0$. For the structure funciton of the real photon the following final expression is obtained:

$$F_2^\gamma(x) = F_2^T(x,0) = \frac{3\alpha}{\pi}\sum_q (e_q)^4 x \Big\{ -1 + 6x(1-x) + \left[x^2 + (1-x)^2\right] ln\frac{Q^2}{x^2p_0^2} + \frac{p_0^4}{2m_\rho^4}\left[x^2 + (1-x)^2 - \frac{8\pi^2}{27}\frac{\langle 0 \mid \frac{\alpha_s}{\pi} G^a_{\mu\nu}G^{a\mu\nu} \mid 0\rangle}{p_0^4x^2}\right] \Big\} \qquad (37)$$

As numerical analysis reveals, this expression is applicable in the interval $0.2 < x < 0.7$. Importantly, the resulting structure function (37) contains two parts. The first two terms, one of them proportional to the logarithm, should be identified with the perturbative part. The dimensionful parameter under the logarithm is p_0^2, corresponding to the integration over hard part of the quark loop. In all previous studies this parameter was simply fitted from experimental data. Furthermore, it is natural to interpret the third term in (37) as the hadronic part. In terms of duality, it contain the soft part of the quark loop and the main nonperturbative term. Numerically, the hadronic part is substantially larger than what one usually has (in the vector dominance model) and is also sensitive to the choice of p_0^2. The expression (37) does not contain new parameters and nothing should be fitted. Nevertheless, the experimental data are described reasonably well in the region of not very large Q^2 where standard evolution of the perturbative loop are not yet essential (see Fig. 4.2). Recently, the problem of evolution of the photon structure function in this approach was studied in [44].

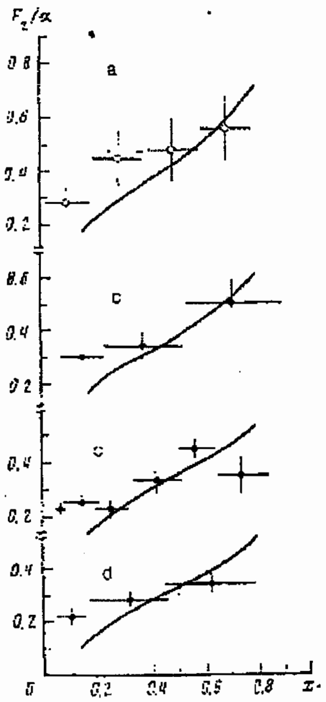

Fig. 4.2 *Comparison of the photon structure function (37)with experimental data from [46] at $Q^2 =$ (a) 23 GeV^2; (b) 9.2 GeV^2; (c) 5.3 GeV^2; (d) 4.3 GeV^2.*

4.3 Calculation of the twist 4 correction

In the suggested framework, one is able to estimate [45] also the twist 4 corrections to the photon structure function. These are scaling violating effects having order of μ^2/Q^2, where μ

is some characteristic hadronic parameter of order of Λ_{QCD}. The knowledge of higher twist corrections is important for comparing with experimental data at small Q^2.

The procedure of extraction of twist 4 corrections, in essence, coincides with the way of calculating the structure function itself. The same main steps should be repeated: OPE calculation for the virtual photon, parametrization of the dispersion relation by comparing with the result of the short-distance calculation at $p^2 \neq 0$, and , finally, analytical continuation to the point $p^2 = 0$. However, now one has to retain all scaling-violating terms of the kind p_0^2/Q^2, m_V^2/Q^2. The most essential and technically difficult element of the calculation is the twist 4 correction to the gluon condensate term in (37). We present the final result:

$$F_2^{T\langle G^2\rangle}(x,p^2) = -\frac{4\pi\alpha}{9}\sum_q (e_q)^4 \frac{\langle 0 \mid \frac{\alpha_s}{\pi} G^a_{\mu\nu} G^{a\mu\nu} \mid 0\rangle}{xp^4}\left(1-\frac{3x^4p^2}{Q^2}\right) . \qquad (38)$$

The expression for the scaling-violating twist 4 correction to the structure function of the real photon can be found in [45]. As numerical analysis reveals, in the intermediate region of x the twist 4 corrections are small even at $Q^2 = 1.5 \div 8$ GeV^2. This fact is important for the selfconsistency of the approach. Meanwhile, when the experimental data in this region will have a few percent accuracy, the higher twist effects should be taken into account.

4.4 Contribution of c quarks to the photon structure

At high energies, a noticeable part of the two-photon cross-section is due to charm production. Summed over final hadronic states, this cross-section is, in first approximation, given by the partonic process $\gamma\gamma^* \to \bar{c}c$. In chapter 3, this process was considered in the case of two real photons. When one of the photons is virtual, the charm production cross section is nothing but a $\bar{c}c$ contribution to the structure functions of the second (real) photon. This part of the structure functions which we denote by $F^c_{1,2}(x)$ is usually subtracted from experimental data, being considered as some sort of a background. In reality, as it was firtsly noticed in [47], the accurate measurement of the charm production cross-section itself would be very important. The point is that the moments of $\sigma(\gamma\gamma^* \to charm)$, or, equivalently the moments of $F^c_{1,2}(x)$, are calculable in QCD with the account of all major nonperturbative corrections. As already discussed in the previous sections, there is no such possibility for two photon-production of light hadrons. The initial correlator in the case of light quarks contains long- distance region at forward scattering , and only its imaginary part at certain region of s corresponding to intermediate $x = Q^2/(Q^2+s)$ can be calculated.

In order to estimate the charm contribution to the photon structure function one has to return to the 4-point correlator (31) with c-quark currents and consider it in the kinematical region $p^2 = 0, q^2 < 0$ in the form of dispersion integral in $s = (p+q)^2$, at some spacelike point $s = \tilde{s} < 0$. The imaginary part is decomposed into the structure functions $F^c_{1,2}$. After taking the nth derivative with respect to $\tilde{s}$ and putting $\tilde{s} = -Q^2$ the dispersion integral is easily transformed into the nth moment of the structure function,

$$M^c_{n(1,2)} \equiv \int_0^1 F^c_{1,2}(x^*)x^{*n-1}dx^* \qquad (39)$$

where $x^* = x(Q^2+4m_c^2)/Q^2$ is a more convenient scaling variable in the case of deep inelastic production a massive state on a massless target.

It is possible to use OPE for $M^c_{n(1,2)}$ directly. The contribution of the unit operator corresponds to the simple 4-point loop (Fig. 4.1,a,b) with massive quarks. The new task is to calculate the gluon condensate contribution. The latter is determined by the diagrams Fig. 4c-d. Using the Fock-Schwinger gauge one reduces very complicated initial expressions to the form of standard one-loop integrals. The latter were calculated with the help of computer algebra " REDUCE". The byproduct of the same calculation at $Q^2 = 0$ is the expression (35).

As an example, we present the obtained result for the gluon condensate contribution to the moments of the structure function F_2^c:

$$M_{n(2)}^{c(G)} = -\frac{3\alpha Q_c^4 \Phi}{16\pi} x_c^{-n} \int_0^{x_c} dx x^n \Big\{ 8(1-v^2)^3 \big[1 - v^2 - (3 - v^2)x\big]$$
$$\times (1-x)^3 \ln\left(\frac{1+v}{1-v}\right) - 16 v^7 (x^4 - 4x^3 + 6x^2 - 4x + 1)$$
$$+\frac{16}{5} v^5 (17x^4 - 62x^3 + 84x^2 - 50x + 11) + \frac{1}{3} v^3 (432x^4 + 320x^3 - 2232x^2 + 1908x - 437)$$
$$-2v(24x^4 - 80x^3 + 72x^2 - 42x + 17) + \frac{1}{v}(16x^2 - 48x + 41)$$
$$+\frac{32}{v}(n+1)x(1-x)^2 + \frac{4}{v}(n+1)(n+2)(1-x)^2 \Big\} \qquad (40)$$

where $x_c = Q^2/(Q^2+4m_c^2)$; $v = \sqrt{1 - 4m_c^2 x/Q^2(1-x)}$ and where the dimensionless parameter Φ is defined in (15). Numerically, the moments with sufficiently large numbers are very sensitive to this contribution (see Fig. 4.3). In duality sense, the revealed situation corresponds to the substantial gluon condensate contribution to the structure functions at large x, i.e. in the resonance region of the $\bar{c}c$ production. In the region of large s (which at fixed $Q^2 = -q^2$ corresponds to small x), i.e. far enough from resonances, the calculation of the imaginary part of the correlator is also possible yielding for the structure functions:

$$F_{1,2}^c(x) = F_{1,2}^{c(0)}(x) + F_{1,2}^{c(G)}(x) \ , \qquad (41)$$

where the unit operator contribution is

$$F_2^{c(0)}(x) = 2x F_1^{c(0)}(x) = \frac{3\alpha Q_c^4}{\pi} ln \left[\frac{Q^2(1-x)}{m_c^2 x}\right] x(1 - 2x + 2x^2) \qquad (42)$$

and the gluon condensate contribution is

$$F_2^{c(G)}(x) = 2x F_1^{c(G)}(x) \simeq \frac{2\alpha Q_c^4}{\pi} \Phi x (9 - 50x + 54x^2 - 16x^4) \ . \qquad (43)$$

One has to notice an essential difference from the photon structure function for light quarks considered above. The latter diverges at small x indicating that OPE is not applicable in this limit. For the charm part of the structure function, the heavy mass of the c quark guarantees that the limit $x \to 0$ is safe. Numerically, the gluon condensate contribution (43) is small as compared with the perturbative part (42). Thus, $F_{1,2}^{c(0)}(x)$ is a very good approximation at large values of c.m. energy $\sqrt{s}$ which is consistent with usual duality approximation.

Summarizing, the charm contribution to the photon structure function is calculable using OPE. The moments of this function are sensitive to the effective c quark interaction with the gluon condensate. Extraction of the gluon condensate density from accurate measurements of these moments, can be suggested as another new way to estimate this important QCD parameter.

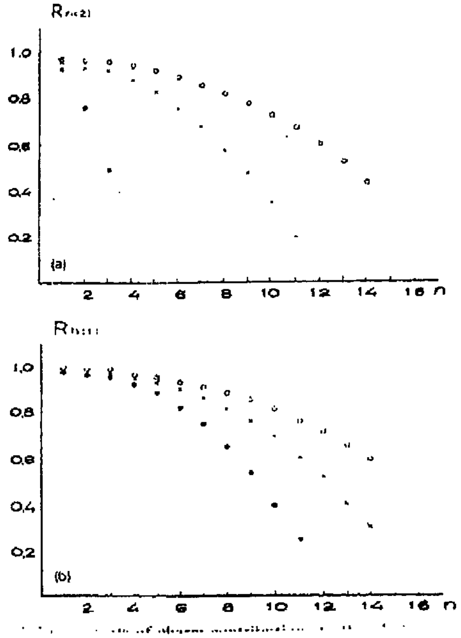

Fig. 4.3 *The influence of the gluon condensate contribution on the charm part of the photon structure functions (a)F_2^c and (b)F_1^c at $Q^2 = 1$ GeV^2 (black circles), 3 GeV^2 (crosses) and 5 GeV^2 (black circles). The plotted ratio R_n is obtained by dividing the moments M_n^c by their perturbative values.*

5 Structure functions of heavy mesons

Traditional applications of QCD to the structure functions of hadrons are concentrated on logarithmic evolution effects within the perturbation theory in the quark-gluon coupling α_s. Initial parton distributions at fixed Q^2 are usually taken from certain models or fitted from experimental data. In order to calculate the parton distributions inside a given hadron directly, one clearly needs noperturbative methods. The most straightforward approach suggested in the framework of the QCD sum rules in [13, 43], reproduces the valence quark distribution in nucleon at intermediate values of Bjorken variable x. The method is based on the OPE of the 4-point correlator. As shown in [48], introducing analogous correlator with heavy quarks, one is able to proceed even further and to calculate the moments of the heavy quark parton distribution inside the heavy meson.

5.1 Derivation of QCD sum rule

Consider the following 4-point correlation function:

$$C_{\mu\nu}(p_1,p_2,q_1,q_2) = -i\int d^4x d^4y d^4z \exp(iq_1x - iq_2y - ip_2z)$$

$$\times\langle 0 \mid T\{\bar{c}i\gamma_5 u(0), \bar{c}\gamma_\mu c(x), \bar{c}\gamma_\nu c(y), \bar{u}i\gamma_5 c(z)\} \mid 0\rangle = C(p_1^2,p_2^2,q_1^2,q_2^2,s,t)g_{\mu\nu} + ... \qquad (44)$$

where the most convenient kinematical structure is retained. For definiteness, we start with the correlator with c quark currents interpolating charmed mesons. The replacement $c \to b$ allows to get analogous results for B mesons.

In (44), the current $\bar{c}i\gamma_5 u$ creates D^0 meson from vacuum. The currents $\bar{c}\gamma_{\mu,\nu}c$ correspond to creation and absorption of a virtual photon by a heavy quark. At fixed spacelike values of p_i^2, q_i^2, $s = (p_1+q_1)^2 = (p_2+q_2)^2$ and at $t = (q_1-q_2)^2 = 0$ the correlator (44) is calculable in terms of OPE. The most important circumstance is the fact that at $t=0$ the hadronic states in the t channel of the correlator are still far enough, at a distance $O(4m_c^2)$. This makes essential difference from the case of the correlator with light-quark currents. Not only the imaginary part of the correlator, but also the correlator itself can be well approximated by the sum of leading OPE terms, including the contributions of the unit operator, the quark condensate and the quark-gluon condensate operators. The gluon condensate contribution was not calculated within adopted accuracy. We relied on analogous results for two- and three-point heavy–light correlators which show that this contribution is numerically unimportant.

The result of the calculation of the invariant amplitude C at $t=0$ obtained with the help of the standard external field technique and in the fixed point gauge for the gluon field is rather complicated. However, it is simplified after making double Borel transformation

$$B_{M^2}f(Q^2) = lim_{Q^2,n\to\infty,Q^2/n=M^2}\frac{(Q^2)^{(n+1)}}{n!}\left(-\frac{d}{dQ^2}\right)^n f(Q^2) \equiv f(M^2) \qquad (45)$$

in variables p_1^2, p_2^2, equating the corresponding independent Borel masses: $M_1^2 = M_2^2 = M^2$ and, finally, differentiating n times over the external variable s. The final answer for the unit operator contribution (the loop diagram of Fig. 5.1a) is obtained in a form of dispersion relation ($q_1^2 = q_2^2 = -Q^2$):

$$\frac{d^n}{ds^n}C^1(M^2,Q^2,s) = \frac{1}{\pi^2}\int ds_1 ds_2 \frac{d^n}{ds^n}\rho(s_1,s_2,Q^2,s)\exp\left(-\frac{s_1+s_2}{M^2}\right), \qquad (46)$$

where the double spectral funciton is symmetrical over s_1 and s_2 :

$$\rho(s_1,s_2,Q^2,s) = \delta(s_1-s_2)\tilde{\rho}(s_1,Q^2,s)\Theta(s_1-m_c^2)\Theta(s_2-m_c^2)\ , \qquad (47)$$

and the expression for the reduced spectral density $\tilde{\rho}$ is presented in [48]. The quark condensate contribution (Fig. 5.1b) and the quark-gluon condensate contribution (Figs. 5.1b and c) are also taken into account.

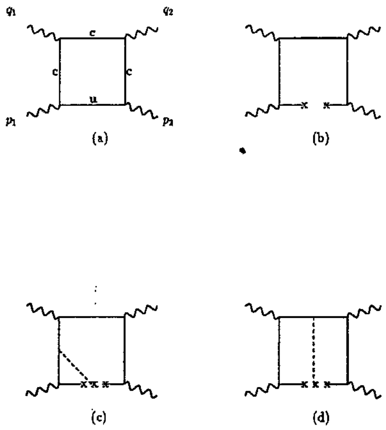

Fig. 5.1 *The diagrams corresponding to (a) the unit operator contribution to the OPE of the correlator (44). Diagrams (b)-(d) contribute to the quark and quark-gluon condensate contributions.*

Furthermore, one represents the correlator (44) in a form of double dispersion relation in p_1^2 and p_2^2:

$$C(p_1^2,p_2^2,q_1^2,q_2^2,s,t=0) = \int\frac{ds_1}{s_1-p_1^2}\int\frac{ds_2}{s_2-p_2^2}\rho(s_1,s_2,q_1^2,q_2^2,s) \qquad (48)$$

The lowest double D meson-pole contribution to the spectral function is

$$\rho^D = -\pi^2 f_D^2\frac{m_D^4}{m_c^2}\delta(p_1^2-m_D^2)\delta(p_2^2-m_D^2)T^c(q^2,s), \qquad (49)$$

where the amplitude T^c is defined via matrix element

$$T_c^{\mu\nu}(q,p) = \int xd^4xe^{iqx}\langle D \mid T\{\bar{c}\gamma_\mu c(x), \bar{c}\gamma_\nu c(0)\} \mid D\rangle = g_{\mu\nu}T_1^c(q^2,s) + ... \qquad (50)$$

corresponding to the virtual photon forward scattering on the c quark of the D meson. (At $p_1^2 = p_2^2 = p^2$ and $t=0$ the kinematics simplifies: $q_1 = q_2 = q$ and $p_1 = p_2 = p$). The D meson coupling constant f_D is defined in a standard way

$$m_c\langle D \mid \bar{c}i\gamma_5 d \mid 0\rangle = f_D m_D^2\ , \qquad (51)$$

Using quark-hadron duality we approximate the continuum of states above D meson, by the quark loop in the region $s_1 > s_0$, $s_2 > s_0$ of the dispersion representation (46). Returning to the amplitude T_c, one easily notices that its imaginary part in s is proportional to the valence c quark distribution in D meson which we denote $c(x, Q^2)$ where $Q^2 = -q^2$ and $x = Q^2/2(p\cdot q)$ is the usual Bjorken variable. Furthermore, at fixed Q^2, the corresponding dispersion integral is easily transformed into an integral over the parton distribution with a certain weight. After differentiation at specially chosen spacelike point $s = m_D^2 - Q^2$ ($Q^2 \ll m_D^2 - m_c^2$ is implied) the integral turns into the $(n-1)-$th moment of $c(x)$. Finally, we equate the Borel transformed and differentiated hadronic representation (48) to the sum of OPE contribuitons. As a result, one obtains the following sum rule for the moments of the heavy quark distribution in a heavy meson:

$$\mathcal{M}_n^c = \int_0^1 dx x^{n-1} c(x, Q^2)$$

$$= \frac{m_c^2}{f_D^2 m_D^4}\Big\{\frac{1}{\pi^2}\int_{m_c^2}^{s_0} ds_1 \exp\left[-\frac{2(s_1 - m_D^2)}{M^2}\right]\frac{(Q^2)^{n+1}}{n!}\frac{d^n}{ds^n}\tilde{\rho}(s_1, Q^2, s)|_{s=m_D^2-Q^2}$$

$$-m_c\langle\bar{q}q\rangle\Big[L^{4/9} + \frac{m_0^2}{2M^2}\Big\{4 - \frac{2m_c^2}{M^2} - \frac{(n+1)}{u}\left(1 + \frac{2m_c^2}{Q^2}\right) + \frac{M^2}{Q^2}\Big[1 + \frac{2}{3u}(n+1)$$

$$-\frac{m_c^2}{Q^2u^2}(n+1)(n+2)\Big]\Big\}\Big]u^{n+1}\exp\left[-\frac{2(m_c^2 - M^2)}{M^2}\right]\Big\} \tag{52}$$

where $u = 1 - \frac{m_D^2 - m_c^2}{Q^2}$.

We use the standard parametrization for the quark-gluon condensate density $\langle\bar{q}\sigma^{\mu\nu}\frac{\lambda^a}{2}g_s G^a_{\mu\nu}q\rangle = m_0^2\langle\bar{q}q\rangle$ in terms of the quark-condensate density. The factor proportional to $L = \{ln(m_c/\Lambda_{QCD})/ln(\mu/\Lambda_{QCD}\}$ accounts for the perturbative renormalization of the quark condensate density.

5.2 Moments of the heavy quark parton distribution

The obtained sum rules (52) do not, in fact, contain new parameters. The effective threshold s_0 =6 GeV^2, together with the coupling constant f_D = 170 MeV and the value of the c quark mass m_c = 1.3 GeV is taken from the analysis of the two-point QCD sum rules for two D meson currents. The numerical analysis of (52) indicates that the moments of the sum rule are applicable at low $n \leq 4$ and at $Q^2 \gg m_D^2 - m_c^2$. This is still the region where $\alpha_s ln[(Q^2 + m_c^2)/\Lambda^2_{QCD}] \ll 1$ and standard perturbative QCD evolution of the moments can be neglected. In the region of their applicability, the sum rules predict very weak dependence of the moments on Q^2. As a reference point we make all numerical predictions at Q^2 = 20 GeV^2 for D-meson (70 GeV^2 for B-meson). At larger Q^2 one has to take into account the evolution of $\mathcal{M}_n^c$.

The first moment calculated from (52) is a very stable function of the Borel parameter M^2 with an average value close to expected $\mathcal{M}_1^c = 1$. It is in fact a nontrivial check of the method because separate OPE contributions add up to give the unit value. At $n > 1$ it is more convenient to work with the ratios of neighboring moments. The obtained moments of c (b) quark-parton distributions in D (B) meson are presented in Table 5.1. The values are close to unit, corresponding to the leading effect [49]–[51] of the heavy quark inside the heavy meson. Note that for b quark the leading effect is numerically more pronounced, as expected. The further details can be found in [48].

Table 1: Moments of the $c(b)$ quark distributions $\mathcal{M}_n^c(\mathcal{M}_n^b)$ in $D(B)$ mesons with various J^P at Q^2 = 20 GeV^2 (Q^2 = 70 GeV^2)

n	$J^P = 0^-, 1^-$	$J^P = 0^+, 1^+$
2	0.85(0.94)	0.65(0.80)
3	0.75(0.89)	0.48(0.68)
4	0.67(0.85)	0.38(0.60)

5.3 Parton distributions depend on the spin-parity of the heavy hadron

The advantage of the presented method is a unique possibility to repeat similar calculation for another heavy meson with different quantum numbers, replacing the relevant currents in the correlator (44). In [52] we calculated the moments of the parton distributions $c(x)$ for mesons with other spin-parities: vector (1^{--}), scalar ($J^{PC} = 0^{++}$), and axial (1^{++}). The obtained results are presented in Table 5.1. Within the accuracy of the method, the structure functions for S-wave (pseudoscalar and vector) heavy mesons have equal lowest moments. The same is valid for P-wave (scalar and axial) heavy mesons. Simultaneously, there is a distinct difference between two groups of mesons. The average part of the meson momentum, carried by the heavy quark, is lower in P-wave mesons. The difference in the sum rules emerges due to the simple fact that the relative sign of condensate contributions and the loop diagram is different in correlators with S and P wave currents. In other words, the interaction with the condensate enhances the leading effect of the heavy quark in the S wave meson and suppresses it in the P-wave meson. The situation resembles two-point sum rules [5] where the interference and interplay of various contributions in the OPE explains why hadrons with very close quantum number and similar flavour content (say, vector and axial mesons) are not alike. From the sum rules similar to (52) one draws an new important conclusion : OPE predicts that the structure functions , or equivalently, the valence parton distributions depend on the spin-parity of the hadron. Furthermore, the sum rules [52], predicting degeneration between structure functions of S wave (pseudoscalar and vector) heavy mesons as well as between structure functions of P wave (scalar and axial) heavy mesons are in complete accord with the expectations of Heavy Quark Effective Theory (see e.g. the review [53]).

5.4 Fragmentation of heavy quarks

Direct experimental test of the results presented above is impossible. However, in various phenomenological approaches, the parton distributions $c(x)$ and $b(x)$ determine the average momentum distribution of the heavy quark not only in the scattering , but also in decay processes of heavy mesons. Our predictions for the heavy quark distributions have been used in theoretical studies of inclusive semileptonic [54] and exclusive B decays [55]. In addition, one may suggest [48] that the ratio of the neighbouring moments of parton distributions for heavy meson is equal to the analogous ratio for the fragmentation functions of the same heavy meson measured in e^+e^--annihilation. This assumption is far more weak than the simple equality of the fragmentation function and distribution function used in phenomenological studies [56, 57]. Strictly speaking, an equality of that kind is not proved for hadrons (see e.g. discussion in [58]). For the quarks at the level of leading logarithms it corresponds to the familiar reciprocity relation [59, 60]. To deal with fragmentation functions measured at high Q^2 one needs first to evolute the moments of the distribution function obtained above. We make use of the fact that in the initial condition for the evolution equations one may safely neglect the contribution of the light valence quark and sea partons in the heavy meson. Applying then the assumption formulated above one reproduces the moments of the fragmentation function of the heavy meson at high Q^2. We stress that the considered fragmentation function is physically complete, including both hard and soft parts. The latter is usually described by a certain model. The suggested approach is able to avoid such division. On the other hand, one may use the obtained moments and factoring out the hard part, extract the model-dependent parameter characterizing the soft fragmentation. Further details of this analysis can be found in [48].

5.5 Regge asymptotics of scattering on heavy mesons

In certain models [61, 62] of soft hadronic processes the concept of parton distributions is used in the framework of Regge-pole phenomenology of high-energy hadron scattering. In these models, the momentum distribution of heavy quarks inside heavy mesons is parametrized in the following form :

$$c(x) = \left(\frac{x}{1-x}\right)^{-\alpha_\psi(0)} \theta(\frac{1}{2} - x) + \left(\frac{1-x}{x}\right)^{-\alpha_\rho(0)} \theta(x - \frac{1}{2}) . \tag{53}$$

where $\alpha_\psi(0)$ and $\alpha_\rho(0)$ are the intercepts of J/ψ and ρ Regge–trajectories. Using the values of moments of $c(x)$ obtained above we have fitted the ansatz (53) with the following results: $\alpha_\rho(0) = 0.2 \div 0.5$ and

$$\alpha_\psi(0) = -(2.0 \div 3.0). \tag{54}$$

The latter parameter is especially important, since one has no independent information on the charmonium trajectory from the physics of soft processes. The intercept $\alpha_\psi(0)$ determines [61, 62] the hadroproduction cross sections of charmed hadrons.

Additional information about intercepts of Regge trajectories, connected with heavy quark-antiquark mesons, is available by considering the scattering of photons on heavy mesons. One uses OPE and dispersion relations in the spirit of the approach described above and compares the result for the moments of the scattering cross sections with their Regge-parametrization. A detailed analysis of this problem is presented in [63]. Two independent ways are considered there. The first one is to use the amplitude of forward photon scattering on quarkonium, and the second one is to consider the virtual photon scattering on a heavy meson. The calculation of the correlation function in both cases is very similar to the procedures presented above and we will not repeat the details. The first method confirms that the intercept α_ψ is within the interval (54). The second method gives uncertain results for this intercept, but turns out to be more useful for estimating the intercept of the b quarkonim trajectory. The following upper limit for the absolute value of this negative quantity is obtained :

$$|\alpha_\Upsilon(0)| \geq 7 \div 8. \tag{55}$$

These estimates have been recently used in [64] in estimating the hadroproduction cross section of B-mesons.

6 Applications of QCD sum rules to the proton spin structure

In the introduction to the previous chapter, it was already mentioned that the four-point correlator may serve as a starting object to calculate the nucleon structure functions. However, in contrast to the correlators of heavy and heavy-light quark currents considered above, the applicability of this direct method is restricted by the region of intermediate values of x, similar to the case of the photon structure function considered in chapter 4. In this limited region of appllicability the major twist-two structure functions of the proton, $F_{1,2}(x)$ and $g_{1,2}(x)$, were estimated [43, 65]. Very convincing is the coinsidence of this parameter-free prediction for g_1, made before the experimental measurement, with recent data [66]. In the first part of this chapter we present the results of the first calculation [67] of twist-two nucleon structure function $h_1(x)$.

Another approach to the direct calculation of structure functions is to use current algebra and to express the integrals over of the parton distributions via hadron couplings with certain quark operators. In this way, in particular, the total part of the quark spin projection in the proton can be expressed in terms of the coupling constant with the flavour singlet axial current. For this kind of matrix elements with zero momentum transfer to the current, a special technique was developed [14, 15] which uses OPE in an external static field with appropriate quantum numbers. The hadronic constant is then obtained in terms of certain induced condensates, corresponding to the vacuum fluctuations of quark and gluon vacuum fields in a presence of external field. This method was applied in [68] to obtain a sum rule for the proton singlet axial coupling.

6.1 The 4-point correlation function

The cross section of the deep inelastic scattering on a given hadron is proportional to the imaginary part of the forward amplitude of virtual photon-hadron scattering. The highly virtual photon is predominantly absorbed and emitted by the same quark and the chirality conservation is evident (Fig. 6.1a). If Drell-Yan process is represented in analogous way, as an imaginary part of the Fig. 6.1b diagram, then virtual photons may interact with different quarks and their chiralities may also be different. Therefore, Drell-Yan process may be used to measure the chirality violating proton structure function $h_1(x)$ introduced and investigated in detail in [69, 70, 71]. It is possible [70] to demonstrate that $h_1(x)$ can be defined as an imaginary part of the forward scattering matrix element defined as

$$T_\mu(p,q,s,s) = \frac{i}{2}\int d^4x e^{iqx} < p,s \mid T\{j_{\mu5}(x), j(0) + j(x), j_{\mu5}(0)\} \mid p,s > \tag{56}$$

where $j_{\mu5}(x)$ and $j(x)$ are axial and scalar currents, respectively.

In order to calculate the structure function $h_1(x)$ by means of the QCD sum rule approach we consider [67] as a starting object the four-point vacuum correlator

$$\Pi_\mu(p,q) = -i\int d^4x d^4y d^4z e^{iqx+ip(y-z)}$$
$$\times < 0 \mid T\{\eta(y), 1/2(j_{\mu5}(x), j(0) + j(x), j_{\mu5}(0)), \bar\eta(z)\} \mid 0 > \tag{57}$$

where η is the three-quark current with proton quantum numbers [12]

$$\eta = \varepsilon^{abc}(u^a C\gamma_\lambda u^b)\gamma_5\gamma_\lambda d^c \tag{58}$$

This correlator corresponds to the amplitude of forward scattering of the current $\eta(x)$ on axial current with transition into scattering on scalar current. The momenta, corresponding to the currents $\eta(x)$ and j_μ, j are p and q, respectively.

As was shown in [13, 43], and already used in chapter 4, the imaginary part of the forward scattering amplitude is determined by small distances in t-channel if p^2 and q^2 are negative and large enough, $\mid p^2 \mid, \mid q^2 \mid \gg \Lambda^2_{QCD}$, and the scaling variable $x = Q^2/2\nu$, $\quad Q^2 = -q^2$, $\quad \nu = p \cdot q$ is not close to the boundary values $x = 0$ and $x = 1$. Therefore, in this region one may use OPE to calculate the amplitude $Im\Pi_\mu(p,q)$. If, in addition, we suppose $\mid p^2 \mid \ll \mid q^2 \mid$ and restrict ourselves to the first term in expansion over p^2/q^2, only twist-two structures will be retained. Since $h_1(x)$ violates chirality, the OPE starts from the operator of dimension 3 - the quark condensate, unlike the case of other twist-two structure functions, where the leading contribution originates from the unit operator. The examples of diagrams corresponding to the OPE of (57) are shown in Fig.6.2. We consider the case when the axial and scalar currents are u -quark currents. The next-to-leading contribution to OPE is given by quark-gluon condensate of dimension 5. For the case of scattering on d-quark both contributions of quark- and quark-gluon condensate vanish. Since the squared electric charge of d-quark is 4 times smaller than that of u-quark, we can safely disregard the higher order OPE contributions of d-quark to the proton structure function $h_1(x)$.

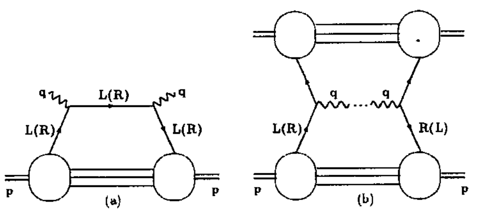

Fig. 6.1: *(a) Deep inelastic lepton-hadron scattering, the chirality of quarks is conserved. Solid lines are quarks, wavy lines are virtual photons, R(L) denote right (left) chirality of quarks; (b) Drell-Yan process with chirality of quarks flipped.*

6.2 QCD sum rule determination of $h_1(x)$.

The amplitude $Im\Pi_\mu(p,q)$ calculated in QCD is equated to the contribution of hadronic states, the proton and the excited states with proton quantum numbers, by means of the dispersion relation in p^2. We are interested in lowest resonance term, which is given by the double pole proton contribution to the dispersion relation in p^2 :

$$\Pi_\mu^{(p)}(p,q) = \lambda_N^2 \frac{1}{(p^2-m^2)^2} \sum_{r,r'} v^r(p) T_\mu^{(p)}(p,q,r,r') \bar{v}^{r'}(p) \tag{59}$$

where m is the proton mass, $v^r(p)$ is the proton spinor with momentum and polarization r, λ_N is the transition constant of proton into quark current

$$< 0 \mid \eta \mid p, r >= \lambda_N v^r(p) \tag{60}$$

and $T_\mu^{(p)}(p,q,r,r')$ is the matrix element (56) nondiagonal in the proton spin. We omit the details of how the spin-tensor structure whose imaginary part is proportional to $h_1(x)$ is separated. The proton contribution we are interested in, is then enhanced by applying the Borel transformation in p^2 and additional differentiation in $1/M^2$ which suppresses the contributions of excited states.

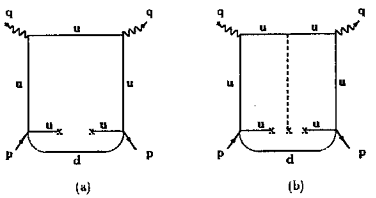

Fig. 6.2: *Examples of diagrams corresponding to the OPE of the four-point correlator Π_μ defined in (57). Wavy lines denote axial or scalar currents, external momenta are shown by arrows, lines with a cross denote the quark vacuum fields, dashed lines denote the vacuum gluon field.*

The following final sum rule for the u-quark part of the structure function was obtained:

$$h_1^u(x) = 2\frac{a}{m\tilde{\lambda}_N^2} e^{m^2/M^2}\{2M^2x[(m^2-M^2)E_1(\frac{W^2}{M^2}) + \frac{W^4}{M^2}e^{-W^2/M^2}]L^{-4/9} + \frac{1}{6}m_0^2 m^2(\frac{1}{x} - \frac{1}{2} - 3x)L^{-8/9}\} \tag{61}$$

where $a = -(2\pi)^2\langle\bar{q}q\rangle = 0.55$ GeV^2, $\tilde{\lambda}_N^2 = 32\pi^4\lambda_N^2 = 2.1$ GeV^6, $E_1(z) = 1 - e^{-z}(1+z)$, and $W^2 = 2.3GeV^2$ is the continuum threshold. The powers of the factor $L = ln(M/\Lambda)/ln(\mu/\Lambda)$ take into account anomalous dimensions of currents in the correlator and operators in the OPE. The numerical values of parameters are taken from two-point sum rules for nucleon mass and magnetic moments [12, 14]. The structure function $h_1(x)$ calculated in [67] is reliable at $0.3 < x \lesssim 0.6$. The lower limit is determined by the requirement on the highest OPE term to remain subdominant. In order to establish the upper limit we use the validity of general inequalities relating $h_1(x)$ with other quark distributions in the nucleon [72] . We estimate the accuracy of this prediction by $\sim 30\%$ at $0.3 < x < 0.6$. As was explained above, the contribution of d quarks is small and the proton structure function

$$h_1(x) \approx (4/9)h_1^u(x). \tag{62}$$

The result for $h_1(x)$ is plotted in Fig. 6.3 where the Regge behaviour with relevant a_1 trajectory is assumed for extrapolation to lower values of x, $0 < x < 0.3$, the region beyound the sum rule validity. We assume that this trajectory is linear and has the same slope as ρ, a_2 trajectories, namely $\alpha' \simeq 1$ GeV^{-2}, Since we disregard the perturbative QCD correction, our results are valid at intermediate $Q^2 \approx 5 - 10$ GeV^2, where deviations from scaling are inessential. We have calculated only two terms in the OPE. Nevertheless, once the method is established, the accuracy may be improved in future taking into account higher order terms of OPE. We estimate the current uncertainty of our calculation at the level of 30-50%. Therefore, having in mind that the similar calculation of the proton spin structure function $g_1(x)$ has reasonable agreement with experiment we consider our estimate of $h_1(x)$ as a reliable and useful guideline for experiments aimed at measuring this new proton structure function.

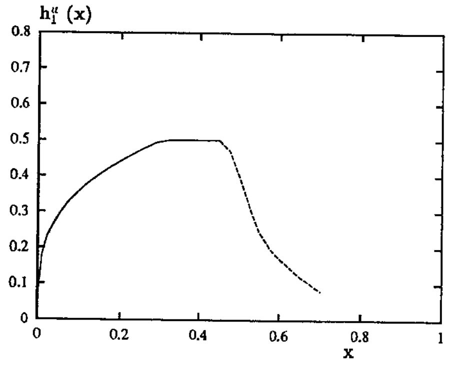

Fig. 6.3:
The u-quark contribution to the proton structure function $h_1(x)$ based on the QCD calculation at intermediate values of x. In the region of low x an extrapolation from the point x =0.3 is done according to the expected Regge behaviour of this structure function. The dashed line shows the values of $1/2(u(x)+g_1^u(x))$ which serve as an upper limit for $h_1^u(x)$ at large x.

6.3 Sum rule for quark spin fraction in deep inelastic scattering

The total fraction of the spin projection of a polarized proton carried by all three light quarks:

$$\Delta\Sigma = \Delta u + \Delta d + \Delta s \ , \tag{63}$$

with the help of current algebra can be expressed in terms of the proton coupling with $SU(3)$ flavour –singlet axial current (see e.g. [73]):

$$2m_p s_\mu \Delta\Sigma = \langle p \mid j^0_{\mu 5} \mid p\rangle = 2m_p s_\mu g^0_A \ , \tag{64}$$

where $j^0_{\mu5} = \sum_q \bar{q}\gamma_\mu\gamma_5 q$, $q = u, d, s$. The corresponding octet and isotriplet couplings of the proton were calculated [74] with the help of QCD sum rules in external axial static field. We calculate the singlet axial constant g^0_A with the same method. The interest to this calculation is stimulated by the relation (63) which allows to link g^0_A with the experimental data on polarized deep inelastic scattering on the nucleon. The calculation contains an important new element, the axial anomaly, which is taken into account explicitly. One considers the polarization operator

$$\Pi(p) = i \int d^4x e^{ip\cdot x}\langle 0 \mid T\{\eta(x), \bar{\eta}(0)\} \mid 0\rangle_A \tag{65}$$

where the proton current η was defined in (58). Index A in (65) means that quarks moove in a constant (static) flavour-singlet axial field A_μ i.e. the QCD Lagrangian contains an additional term $\Delta L = j^0_{\mu5}A^\mu$. We are interested in the term proportional to the first power in A_μ in the expansion of the polarization operator in external field. The idea is to use the dispersion relation for Π_μ and to isolate the lowest double-pole term of the proton. The residue of this pole is proportional to the combination of coupling constants $\lambda_N^2 g^0_A$ where the coupling λ_N is defined above in (60), and was determined from corresponding two-point sum rules in [12].

On the other hand, the correlator Π_μ can be calculated at $p^2 < 0$ using OPE. We present the final sum rule for the axial constant after Borel transformation and applying additional differential operator in M^2 [14] aimed to suppress the contribuitons of nondiagonal transitions of proton into excited states in the dispersion relation:

$$g^0_A = -1 + \frac{8}{9}\frac{1}{\lambda_N^2}(1 - M^2\frac{d}{dM^2})\exp\left(\frac{m^2}{M^2}\right)\left[6\pi^2 f_0^2 M^4 E_1\left(\frac{s_0}{M^2}\right)L^{-\frac{4}{9}}\right.$$
$$\left. +14\pi^2 h_0^2 M^2 E_0\left(\frac{s_0}{M^2}\right)L^{-\frac{8}{9}} + (2\pi)^4\langle\bar{q}q\rangle^2 L^{\frac{4}{9}}\right] \tag{66}$$

where $E_0(x) = 1 - e^{-x}$, $E_1(x) = 1 - (1+x)e^{-x}$, factors proportional $L = ln(M/\Lambda)/ln(\mu/\Lambda)$ account for anomalous dimensions. This sum rule contains, apart from contributions of the loop diagram (unit operator) and square of quark-condensate (four-quark operators), new types of condensates induced by the external axial field A. The densities f_0 and h_0 determine the following vacuum averages:

$$\langle 0 \mid \bar{q}\gamma_\mu\gamma_5 q \mid 0\rangle_A = f_0^2 A_\mu \qquad (d=3)\ , \tag{67}$$

$$\langle 0 \mid \bar{q}\gamma_\mu\frac{1}{2}\lambda^a\tilde{G}^a_{\mu\nu}q \mid 0\rangle_A = h_0^2 A_\nu \qquad (d=6)\ . \tag{68}$$

We neglect the third possible $d = 5$ vacuum average of purely gluonic operator which has small short-distance coefficient in OPE. The axial constant g^0_A is thus expressed via two unknown but universal parameters.

As usual, one has to try to obtain them from other sum rules with known hadronic part. One possibility is to study the two-point sum rules of the corresponding currents , e.g. for f_0 one may use the relation

$$\langle 0 \mid \bar{q}\gamma_\mu\gamma_5 q \mid 0\rangle_A = lim_{k\to 0}\, i\int d^4x e^{ikx}\sum_{q_i=u,d,s}\langle 0 \mid T\{\bar{q}_i\gamma_\nu\gamma_5 q_i(x), \bar{q}\gamma_\mu\gamma_5 q(0)\} \mid 0\rangle A_\nu$$
$$\equiv \Pi_{\mu\nu}(k)A_\nu \tag{69}$$

Since the flavour-singlet axial currents enter this polarization operator, one has to take into account the axial anomaly. Therefore, the situation for condensates generated by singlet currents is fundamentally different from the ones corresponding to isotriplet or octet currents. Using the well-known expression for the anomaly one gets for the longitudinal part of the two-point correlator (69) the following relation:

$$q_\mu q_\nu \Pi_{\mu\nu}(q) = i\frac{\alpha_s}{4\pi}\int d^4x e^{iqx}\langle 0 \mid T\{\frac{3\alpha_s}{4\pi}G^a_{\alpha\beta}(x)\tilde{G}^a_{\alpha\beta}(x)$$
$$+2im_s\bar{s}\gamma_5 s(x), G^a_{\alpha\beta}(0)\tilde{G}^a_{\alpha\beta}(0)\} \mid 0\rangle \equiv -q^2\Pi_L(q^2), \tag{70}$$

Using dispersion relation for this amplitude and estimating contribution of the lowest resonance η' with the help of usual two-point sum rule technique and the contribution of higher states with the help of quark-hadron duality, one may estimate the value $\Pi_L(0)$ and, according to the relation (69), the coupling f_0. The coupling h_o is estimated in a similar manner. However, the sum rule for the correlator (70) turns out to be inconsistent: the hadronic part doesnt fit the OPE. Especially destabilizing effect is induced in this sum rule by the strange quark contribution. The fact that longitudinal part of the correlator of two flavour-singlet axial currents manifests breaking of OPE was anticipated from studues of other nonperturbative aspects of QCD (e.g. instantons). However we found [68] analogous breaking of OPE also in the transverse part of the correlator $\Pi_{\mu\nu}$. Higher terms of OPE or, more probably, other nonperturbative (nonlocal) effects are important in the virtuality region $1 \div 2$ GeV^2. In this respect the axial channel is similar to the flavour-singlet pseudoscalar channel 0^{-+} where the massive η' state cannot be interpolated by usual local OPE. The physical consequence of the breakdown of OPE in the singlet axial channel is that one should expect a strong mixing between f_1 mesons.

Theoretically, the study of the correlator (69) demands methods beyond the usual OPE. In this respect the problem of g_A^0 -coupling can be inverted. Using experimental data (yielding currently $g_A^0 = \Sigma \simeq 0.4 \div 0.5$, see e.g. review [75]) one may use the sum rule (66) to constraint the matrix elements (67) and (68), and consequently , the correlator (69).

7 Light-cone sum rules for heavy-light form factors and exclusive decays

In various applications of QCD sum rules considered above, the key element is the Wilson expansion of the T-product of currents at small distances. In this chapter we consider an alternative to this method, an expansion near the light-cone in terms of nonlocal operators, the matrix elements of which are given by hadron wave functions of increasing twist. As one advantage, this formulation allows to incorporate additional information about the Euclidean asymptotics of correlation functions in QCD for arbitrary external momenta. From practical point of view, this approach is especially useful for exclusive hadronic processes involving among other hadrons a light hadron or a real photon. We will concentrate here on processes involving both heavy and light mesons, including form factors of weak $B \to \pi$ and $D \to \pi$ transitions [76], strong couplings $B^*B\pi$ and $D^*D\pi$ [77] and various weak radiative decays of heavy mesons [78]. Among other practical applications, we present the result [79] of extraction of the CKM parameter V_{ub} from $B \to \pi e \nu$ decay width.

7.1 Operator product expansion near the light-cone

For definiteness, we focus on the correlation function which will later be used to evaluate the heavy-to-light form factor $B \to \pi$ and the strong $B^*B\pi$ coupling:

$$\begin{aligned} F_\mu(p,q) &= i \int d^4x e^{ipx} \langle \pi(q) \mid T\{\bar{u}(x)\gamma_\mu b(x), \bar{b}(0) i\gamma_5 d(0)\} \mid 0\rangle \\ &= F(p^2,(p+q)^2) q_\mu + \tilde{F}(p^2,(p+q)^2) p_\mu \,. \end{aligned} \tag{71}$$

With the pion on mass-shell, $q^2 = m_\pi^2$, the correlation function (71) depends on two invariants, p^2 and $(p+q)^2$. We set $m_\pi = 0$ everywhere.

In the Euclidean region where both p^2 and $(p+q)^2$ are negative and large, the b quark is far off-shell. Substituting, as a first approximation, the free b-quark propagator into eq. (71) one readily obtains

$$\begin{aligned} F_\mu(p,q) &= i \int \frac{d^4x\, d^4k}{(2\pi)^4(m_b^2 - k^2)} e^{i(p-k)x} \left(m_b \langle \pi(q) | \bar{u}(x)\gamma_\mu\gamma_5 d(0) | 0\rangle \right. \\ &\quad \left. + k^\nu \langle \pi(q) | \bar{u}(x)\gamma_\mu\gamma_\nu\gamma_5 d(0) | 0\rangle \right) \,. \end{aligned} \tag{72}$$

This contribution is depicted diagramatically in Fig. 1a.

Short-distance expansion of the first matrix element of (72) in terms of local operators,

$$\bar{u}(x)\gamma_\mu\gamma_5 d(0) = \sum_n \frac{1}{n!} \bar{u}(0)(\overleftarrow{D} \cdot x)^n \gamma_\mu\gamma_5 d(0) \,, \tag{73}$$

and integration over x and k yield

$$F_\mu(p,q) = i \frac{m_b}{m_b^2 - p^2} \sum_{n=0}^{\infty} \frac{(2p\cdot q)^n}{(m_b^2 - p^2)^n} M_n q_\mu \,, \tag{74}$$

where

$$\langle \pi(q) | \bar{d}\, \overleftarrow{D}_{\alpha_1} \overleftarrow{D}_{\alpha_2} \ldots \overleftarrow{D}_{\alpha_n} \gamma_\mu\gamma_5 u | 0\rangle = (i)^n q_\mu q_{\alpha_1} q_{\alpha_2} \ldots q_{\alpha_n} M_n + \ldots$$

has been used, D being the covariant derivative. One now encounters the following problem. If the ratio

$$\tilde{\xi} = 2(p \cdot q)/(m_b^2 - p^2) = ((p+q)^2 - p^2)/(m_b^2 - p^2) \tag{75}$$

is finite one must keep an infinite series of matrix elements of local operators in eq. (74). All of them give contributions of the order $1/(m_b^2 - p^2)$ in the heavy quark propagator, differing only by powers of the dimensionless parameter $\tilde{\xi}$. Therefore, short-distance expansion of eq. (72) is useful only if $\tilde{\xi} \to 0$, i.e. for $p^2 \simeq (p+q)^2$ or, equivalently, $q \simeq 0$. In this case, which in fact effectively corresponds to the scattering on static external field considered in the previous chapter (see also [80, 77]), the series in eq. (74) can be truncated after a few terms involving only a small number of unknown matrix elements M_n. However, for general momenta with $p^2 \neq (p+q)^2$ one has to sum up the infinite series of matrix elements of local operators in some way.

This formidable task can be solved by using techniques developed for hard exclusive processes in QCD [16, 81]. Returning to the initial expression (71) for the correlation function one expands the T-product of currents near the light-cone $x^2 = 0$. In a first step this leads to the same approximation (72) involving vacuum-to-pion transition matrix elements of nonlocal operators composed of light quark fields at light-like separation. These matrix elements are expanded in x and at $x^2 \simeq 0$ reexpressed in terms of pion wave functions with given twist. For the present discussion it is again sufficient to focus on the first term in (72) proportional to m_c. In leading twist one has

$$\langle \pi(q) | \bar{u}(x) \gamma_\mu \gamma_5 P \exp\{ig_s \int_0^1 d\alpha\, x_\mu A^{a\mu}(\alpha x) \lambda^a/2\} d(0) | 0 \rangle = -i q_\mu f_\pi \int_0^1 du\, e^{iuqx} \varphi_\pi(u) \,, \tag{76}$$

where the wave function φ_π represents the distribution in the fraction u of the light-cone momentum $q_0 + q_3$ of the pion carried by a constituent quark. The path-ordered exponential gauge factor ensures gauge invariance.

Substituting (76) in (72) and integrating over x and k one finds for the invariant function F :

$$F(p^2, (p+q)^2) = m_b f_\pi \int_0^1 \frac{du\, \varphi_\pi(u)}{m_b^2 - (p+uq)^2} + \dots \,, \tag{77}$$

where the ellipses represent contributions of higher twists and multicomponent wave functions. The leading three-particle wave function enters in connection with gluon emission by the heavy quark line as shown in Fig. 1b. This contribution is included in the calculations of refs. [76, 77] as well as two-particle wave functions up to twist 4. The perturbative $O(\alpha_s)$ corrections indicated in Fig. 1c are not yet calculated. Their calculation in future may considerably increase the accuracy.

Comparing (74) and (77) one sees that the infinite series of matrix elements of local operators encountered before in (74) is effectively replaced by hadronic wave functions. These universal functions describe the long-distance dynamics similarly as the universal vacuum condensates appearing in the sum rule variant based on short-distance expansion.

Asymptotically, that is at $\mu \to \infty$, QCD perturbation theory implies for the lowest twist 2 wave function $\varphi_\pi(u, \infty) = 6u(1-u)$. However, at the physical scale $\mu \sim m_b$, at which the OPE is applied to the correlation function (71), important nonasymptotic effects are to be expected. These are parameterized by the coefficients $a_i(\mu)$ in the following series of Gegenbauer polynomials:

$$\varphi_\pi(u, \mu) = 6u(1-u)[1 + \sum_{i=2,4,..} a_i(\mu) C_i^{3/2}(2u-1)] \,. \tag{78}$$

Although the input values of the coefficients $a_i(\mu)$ at a fixed low momentum scale μ are unknown, the scale-dependence of $a_i(\mu)$ is dictated by the renormalization group. From what is said above, it is also clear that $a_i(\mu) \to 0$ for $\mu \to \infty$. Over the years a great deal has been learned about these wave functions. They have been classified up to twist 4 and the asymptotic form has been determined. Also various nonasymptotic corrections have been estimated from QCD sum rules for light hadrons. In our calculation we have used the wave functions given in [84].

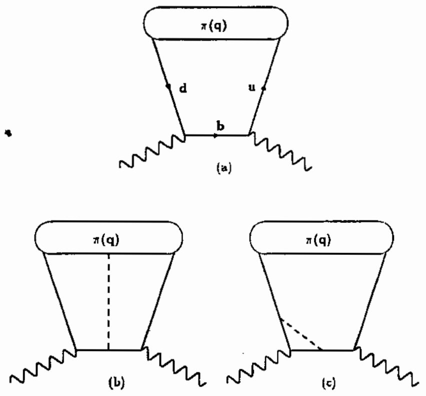

Fig. 7.1: *QCD diagrams contributing to the correlation function (1) and involving (a) quark-antiquark light-cone wave functions, (b) three-particle quark-antiquark-gluon wave functions, and (c) perturbative $O(\alpha_s)$ corrections. Solid lines represent quarks, dashed lines gluons, wavy lines are external currents.*

7.2 Calculation of $B \to \pi$ and $D \to \pi$ form factors

The matrix element of the $B \to \pi$ transition can be written in terms of two independent form factors f^+ and f^-:

$$\langle \pi(q) \mid \bar{u}\gamma_\mu b \mid B(p+q) \rangle = 2f^+(p^2) q_\mu + [f^+(p^2) + f^-(p^2)] p_\mu \,, \tag{79}$$

where $p+q$ and q denote the initial and final state four-momenta, respectively, and $\bar{u}\gamma_\mu b$ is the relevant quark vector current.

The form factor f^+ enters the dispersion relation for the invariant amplitude F of the correlator (71) through the ground state B–meson contribution:

$$F(p^2, (p+q)^2) = \frac{2 f_B m_B^2 f^+(p^2)}{m_b(m_B^2 - (p+q)^2)} + \int_{s_0}^{\infty} ds \frac{\rho^h(p^2, s)}{s - (p+q)^2} \,, \tag{80}$$

where the B meson decay constant f_B is defined similar to f_D in (51). The integral in (80) over the spectral density ρ^h takes into account the contributions from the excited states and the continuum with B meson quantum numbers, s_0 denoting the effective mass threshold. Using the result (77) for the invariant amplitude F obtained in the previous section and applying Borel transformaiton (5.1) to eq. (80) one finds the following sum rule:

$$
\begin{aligned}
f^+(p^2) = & \frac{f_\pi m_b^2}{2 f_B m_B^2} \exp\left(\frac{m_B^2}{M^2}\right) \left\{ \int_\Delta^1 \frac{du}{u} exp\left[-\frac{m_b^2 - p^2(1-u)}{uM^2}\right] \right. \\
& \times \left(\varphi_\pi(u) + \frac{\mu_\pi}{m_b}\left[u\varphi_p(u) + \frac{\varphi_\sigma(u)}{3}\left(1 + \frac{m_b^2 + p^2}{2uM^2}\right)\right] - \frac{4 m_b^2 g_1(u)}{u^2 M^4} \right. \\
& \left. \left. + \frac{2}{uM^2}\int_0^u g_2(v)dv \left(1 + \frac{m_b^2 + p^2}{uM^2}\right)\right) + f_G^+(p^2, M^2) + \ldots, \right. \qquad (81)
\end{aligned}
$$

Here, φ_p, φ_σ, and $\varphi_{3\pi}$ are twist-3 pion wave functions. The ellipses denote contributions of higher twist. The contributions of twist 4 are given explicitly in [76, 77].

The analogous sum rule for the $D \to \pi$ form factor is obtained from the above by formally changing $b \to c$ and $\bar{B} \to D$ with corresponding rescaling of all anomalous dimension factors appearing e.g. in the higher twist wave functions and in nonasymptotic corrections to the lowest twist wave functions. Earlier, the light-cone sum rule for f^+ at $p^2 = 0$ with leading twist accuracy was derived in [82] (see also [83]).

Importantly, the numerical values to be substituted for m_b, f_B and s_0 are interrelated by the QCD sum rule for the two-point correlation function $\langle 0 \mid T\{j_5(x), j_5^+(0)\} \mid 0 \rangle$, $j_5 = \bar{b}i\gamma_5 u$. We use the set $f_B = 140$ MeV , $m_b = 4.7$ GeV, $s_0 = 35$ GeV^2 which satisfies this two-point sum rule without $O(\alpha_s)$ corrections in consistency with the neglect of $O(\alpha_s)$ corrections in the sum rule for $f_B f_B^+$.

The maximum momentum transfer p^2 at which the sum rule (81) is applicable is estimated to be about 1 GeV^2 for D mesons and 15 GeV^2 for B mesons. The resulting form factors $f_B^+(p^2)$ and $f_D^+(p^2)$ are plotted in Fig. 7.2. The dependence of eq. (81) on the Borel parameter M^2 is rather weak in the range where the twist-4 and the continuum contributions are less than 10% and 30%, respectively [76]. For definiteness, we have taken $M^2 = 10$ GeV^2 for the $B \to \pi$ and $M^2 = 4$ GeV^2 for the $D \to \pi$.

7.3 Light-cone sum rules for strong couplings of heavy mesons

Next we describe how the relation (77) can be turned into a sum rule for the strong $B^*B\pi$ coupling constant. The key idea is to write a double dispersion integral for the invariant function F:

$$
\begin{aligned}
F(p^2,(p+q)^2) = & \frac{m_B^2 m_{B^*} f_B f_{B^*} g_{B^*B\pi}}{m_b(p^2 - m_{B^*}^2)((p+q)^2 - m_B^2)} \\
& + \int \frac{\rho^h(s_1,s_2)ds_1 ds_2}{(s_1 - p^2)(s_2 - (p+q)^2)} + \int \frac{\rho_1^h(s_1)ds_1}{s_1 - p^2} + \int \frac{\rho_2^h(s_2)ds_2}{s_2 - (p+q)^2} . \qquad (82)
\end{aligned}
$$

Here, the first term arises from the ground state contribution and contains the $B^*B\pi$ coupling defined by the on-shell matrix element

$$
\langle B^{*-}(p)\pi^+(q) \mid \bar{B}^0(p+q)\rangle = -g_{B^*B\pi} q_\mu \epsilon^\mu . \qquad (83)
$$

while the spectral function $\rho^h(s_1,s_2)$ represents higher resonances and continuum states in the B^* and B channels. f_{B^*} is the coupling of B^* meson to the vector $\bar{u}\gamma_\mu b$ current. The additional single dispersion integrals are due to necessary subtractions. Then, considering p^2 and $(p+q)^2$ as independent variables, using light-cone OPE result (77) for F and applying the Borel operator (45) to eq. (82) with respect to both p^2 and $(p+q)^2$, we obtain the sum rule

$$
f_B f_{B^*} g_{B^*B\pi} = \frac{m_b^2 f_\pi}{m_B^2 m_{B^*}} e^{\frac{m_B^2 + m_{B^*}^2}{2M^2}} \left\{ M^2 [e^{-\frac{m_b^2}{M^2}} - e^{-\frac{s_0}{M^2}}]\varphi_\pi(1/2) + \ldots \right\} \qquad (84)
$$

with $M^2 = M_1^2 M_2^2/(M_1^2 + M_2^2)$. The contributions from heavier states are now exponentially suppressed, while the subtraction terms depending only on one of the variables, p^2 or $(p+q)^2$, vanish.

Since M_1^2 and M_2^2 are expected to be quite similar in magnitude, the coupling constant $g_{B^*B\pi}$ is determined by the value of the pion wave function at $u \simeq 1/2$, that is by the probability for the quark and the antiquark to carry equal momentum fractions in the pion. This interesting feature is shared by the sum rules for many other important hadronic couplings involving the pion. As already pointed out, the quantity $\varphi_\pi(1/2)$ is considered to be a universal nonperturbative parameter, similar to quark and gluon condensates in the standard approach. It may be determined from suitable sum rules in which the phenomenological part is known experimentally. We take the value $\varphi_\pi(1/2)$ =1.2±0.2 obtained from the light-cone sum rule for the pion-nucleon coupling [84]. For the remaining parameters we use the same input values as in the calculation of the form factor f_B^+. In addition, we take $f_{B^*} = 160$ MeV as determined from the corresponding two-point sum rule. With this choice, we obtain

$$
g_{B^*B\pi} = 29 \pm 3 . \qquad (85)
$$

The uncertainty indicates the variation of $g_{B^*B\pi}$ in the corresponding interval 6 GeV$^2 < M^2 <$ 12 GeV^2, where the higher state contributions are less than 30% and the twist-4 corrections do not exceed 10% . The sum rule for $g_{B^*B\pi}$ given in (84) is easily converted into a sum rule for the coupling $g_{D^*D\pi}$ by replacing b with c, B with D, and B^* with D^*. Using the relevant $D-$channel parameters and the optimal interval 2 GeV$^2 < M^2 < 4$ GeV^2, one finds

$$
g_{D^*D\pi} = 12.5 \pm 1.0 . \qquad (86)
$$

The above prediction can be directly tested experimentally in the decay $D^* \to D\pi$. Eq. (86) implies the decay width $\Gamma(D^{*+} \to D^0\pi^+) = 32 \pm 5$keV, which is well below the current experimental upper limit [85, 86] $\Gamma(D^{*+} \to D^0\pi^+) < 89$ keV.

The dependence on the pion wave function disappears in the limit $q \to 0$ as can be seen from (77) because of the normalization condition $\int_0^1 du\varphi_\pi(u) = 1$. This is just the limit where the correlation function (71) can be treated in short-distance expansion by using the external field method.

The couplings $g_{B^*B\pi}$ fix the normalization of the form factors of the heavy-to-light transitions $B \to \pi$, in the pole-model description

$$
f_B^+(p^2) = \frac{f_{B^*} g_{B^*B\pi}}{2m_{B^*}(1 - p^2/m_{B^*}^2)} . \qquad (87)
$$

which is valid at higher values of p^2 close to the kinematical threshold of the soft pion. At intermediate p^2 as can be seen from Fig. 7.2 this description successfully matches the light-cone sum rule prediction. Analogous approximation with the coupling (86) is obtained for the

$D \to \pi$ form factor. Summarizing, the approach presented allows for the first selfconsistent calculation of the heavy-to-light form factor in the whole kinematical region. Interestingly, this task is beyound the abilities of other nonperturbative methods, including HQET and even lattice calculations, at least in their present status.

The accuracy of the light-cone sum rule method is conservatively estimated to be around 20-30%. Currently, the main sources of uncertainty are our limited knowledge of the nonasymptotic terms in the wave functions and the lack of the perturbative α_s-corrections to the correlation function (96).

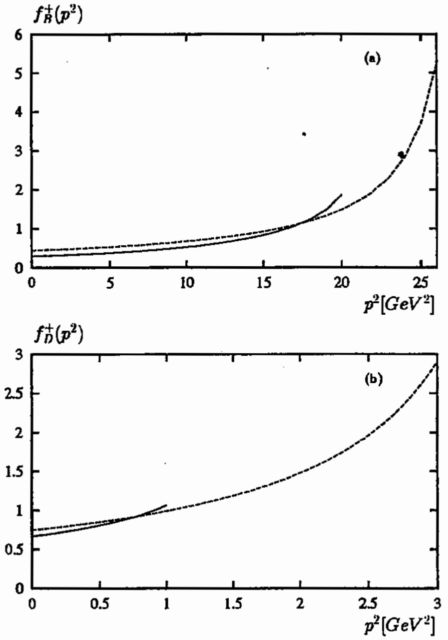

Fig. 7.2. *The form factors for the transitions (a) $B \to \pi$ and (b) $D \to \pi$ as predicted by light-cone sum rules (solid) in comparison to the single-pole approximation (dashed) with the normalization constants $g_{B^*B\pi}$ and $g_{D^*D\pi}$, respectively, calculated by the same method.*

7.4 Extracting V_{ub} from $B \to \pi l \nu$ decay

Recently, the CLEO collaboration [87] announced the first measurement of the decay $B \to \pi l \nu$ $(l = e, \mu)$. This process plays a very important role for the determination of the CKM parameter V_{ub}. The decay amplitude is completely determined by the form factor $f_B^+(p^2)$ which was calculated as explained above. With this result one easily calculates the total width using well-known kinematical formulae. The form factor depicted in Fig. 7.2a yields [79]

$$\Gamma(B^0 \to \pi^- l^+ \nu_l) = 8.1\ |V_{ub}|^2\ \mathrm{ps}^{-1}\ . \tag{88}$$

Experimentally, combining the preliminary CLEO result, $BR(B^0 \to \pi^- l^+ \nu_l) = (1.63 \pm 0.46 \pm 0.34) \cdot 10^{-4}$ [87], with the world average of the B^0 lifetime [30], $\tau_{B^0} = 1.57 \pm 0.05$ ps, one obtains

$$\Gamma(B^0 \to \pi^- l^+ \nu_l) = (1.04 \pm 0.37) \cdot 10^{-4}\ \mathrm{ps}^{-1}\ , \tag{89}$$

Comparison of (88) with (89) yields

$$|V_{ub}| = 0.0036 \pm 0.0006\ . \tag{90}$$

The theoretical uncertainty which we estimate conservatively to be less than 20% is not included in (90).

In the case of the Cabibbo suppressed D meson decay $D^0 \to \pi^- l^+ \nu_l$ we predict

$$\Gamma(D^0 \to \pi^- l^+ \nu_l) = 0.156\ |V_{cd}|^2\ \mathrm{ps}^{-1} = (7.6 \pm 0.2) \cdot 10^{-3}\ \mathrm{ps}^{-1}\ , \tag{91}$$

where we have substituted $|V_{cd}| = 0.221 \pm 0.003$ [30].

This should be compared with the experimental result

$$\Gamma(D^0 \to \pi^- e^+ \nu_e) = (9.4^{+5.5}_{-2.9}) \cdot 10^{-3}\ \mathrm{ps}^{-1}, \tag{92}$$

derived from the branching ratio $BR(D^0 \to \pi^- e^+ \nu) = (3.9^{+2.3}_{-1.2}) \cdot 10^{-3}$ and the lifetime $\tau_{D^0} = 0.415 \pm 0.004$ ps [30].

We see that the CKM-suppressed exclusive semileptonic widths are not yet measured precisely enough to really challenge theory.

7.5 Use of the photon light-cone wave function

Rare decays of B-mesons, such as the recently observed processes $B \to K^* \gamma$ [88] are becoming an important tool for studying new forces beyond the standard model. The interest in these decays stems from the fact that they occur only through loops and are therefore particularly sensitive to "new physics". Besides the $B \to K^* \gamma$ exclusive transition, there are also $B \to \rho\gamma$ processes. The latter modes are sensitive to other CKM matrix elements (and possibly to other new physics) and may show large CP-violation. It is generally believed that the short distance penguin mechanism dominates the exclusive decays. The corresponding matrix element has been calculated with various methods including the light-cone sum rules [90] which yield similar results. However, besides the matrix element of the penguin operator, there are certain long-distance contributions to decay amplitudes which are usually investigated in phenomenological way. Such estimates can give at best an order of magnitude estimate.

As an example of application of the QCD light-cone sum rules we present here the results [78] of calculation of one of these long-distance effects. For definiteness, we will consider the decay $B \to \rho\gamma$, although other decays can be treated in similar ways. Fig. 7.3 represents schematically the mechanism. These are diagrams which do not involve loops of heavy quarks but just the ordinary four-quark weak interaction. In these graphs, the spectator light quark participates in the weak annihilation. Because of the CKM matrix elements, this mechanism is negligible for $B \to K^* \gamma$ but important for other exclusive radiative decays.

A perturbative evaluation of these diagrams is completely inapplicable due to the almost on-shell propagation of the light quark which implies the use of a poorly understood 'constituent' quark mass.

The relevant effective Hamiltonian for the decay $B^- \to \rho^- \gamma$ is

$$\mathcal{H}_W = \frac{G}{\sqrt{2}} V_{ub} V_{ud}^* a_1 (\bar{d} L_\mu u)(\bar{u} L^\mu b) , \tag{93}$$

with $a_1 = c_1 + c_2/3$, a combination of familiar short-distance coefficients $c_{1,2}$, $L_\mu = \gamma_\mu(1-\gamma_5)$. Similarly, the combination $a_2 = c_2/3 + c_1$ multiplies the corresponding operator for $B^0 \to \rho^0 \gamma$. Following the phenomenological ansatz [91], the coefficients a_1, a_2 are extracted from two-body non-leptonic decays assuming factorization. We also neglect the photon emission from the final state quarks invoking the well known helicity arguments: being factorized, the matrix element of (93) is in this case proportional to the light quark masses.

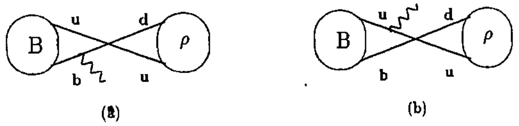

Fig. 7.3: *Weak annihilation mechanism for the decay $B^- \to \rho^- \gamma$. Two additional diagrams with photon emission from the final state quarks are not shown.*

The matrix element corresponding to diagrams in Figs. 7.4a,b can be written as

$$\langle \rho^- \mid H_W \mid B^- \rangle_\gamma = \frac{G}{\sqrt{2}} V_{ub} V_{ud}^* a_1 f_\rho m_\rho \epsilon_\rho^\mu \langle 0 \mid (\bar{u} L_\mu b) \mid B^- \rangle_\gamma , \tag{94}$$

where we used $\langle \rho^- \mid (\bar{d} L^\mu u) \mid 0 \rangle = f_\rho m_\rho \epsilon_\rho^\mu$, denoting by f_ρ and ϵ_ρ the decay constant and the polarization vector of the charged ρ-meson. It then remains to calculate the matrix element

$$\langle 0 \mid (\bar{u} L^\mu b) \mid B^- \rangle_\gamma = -A_{PC} \varepsilon_{\mu\tau\lambda\sigma} p^\tau \epsilon^\lambda q^\sigma + i A_{PV} [q_\mu (\epsilon \cdot p) - \epsilon_\mu (p \cdot q)] , \tag{95}$$

which describes the annihilation of B^- into the current $\bar{u} L^\mu b$ with momentum p accompanied by the emission of a real photon with momentum q and polarization vector ϵ in terms of two invariant, parity conserving and parity-violating amplitudes. The light quark mass is neglected. We use QCD sum rules in order to calculate the matrix element (95). Since the photon emission from the light quark takes place at large distances, the use of standard QCD sum rules [5] based on the local operator product (OPE) expansion is not sufficient. Rather, one should use a light-cone expansion. It will involve the hadronic wave functions on the light-cone which encode the photon emission by a quark-antiquark pair at light-like separation in close analogy to the pion wave function considered above. The photon light-cone wave function was introduced in ref. [92] for calculating the amplitude of weak radiative decay $\Sigma \to \rho\gamma$ and used later in ref. [84] to evaluate the nucleon magnetic moments. We introduce the relevant correlation function

$$\Pi_\mu(p,q) = i \int d^4x e^{ipx} \langle 0 | T\{\bar{u}(x) L_\mu b(x), \bar{b}(0) i\gamma_5 u(0)\} | 0 \rangle_{F(q)} . \tag{96}$$

in the external electromagnetic field $F_{\alpha\beta}(q,x) = i(\epsilon_\beta q_\alpha - \epsilon_\alpha q_\beta) e^{iqx}$ with momentum q and polarization vector ϵ. The function Π_μ can be decomposed into parity conserving and a parity violating invariant amplitudes Π_{PC} and Π_{PV} corresponding to the parametrization in (95). In the region $(p+q)^2 < 0$ and at $p^2 = m_\rho^2 << m_b^2$, the heavy b-quark is far off-shell. In particular,

photon emission from the heavy b-quark takes place perturbatively. The accompanying light-quark propagator may then be described by the local OPE containing as a first approximation the free propagation and in the next orders, interaction with quark and quark-gluon vacuum condensates. The corresponding contributions to the correlation function are depicted in Figs. 7.4a, b and c, respectively. As far as photon emission from the light u-quark is concerned, after contracting the b-quark line, one is left with matrix element

$$\Pi_\mu(p,q) = i \int \frac{d^4x\, d^4k}{(2\pi)^4 (m_b^2 - k^2)} e^{i(p-k)x} \langle 0 | \bar{u}(x) L_\mu (m_b + \not{k}) \gamma_5 u(0) | 0 \rangle_{F(q)} . \tag{97}$$

The diagram Fig. 7.4d describes only the short-distance part of this matrix element corresponding to the photon emission from freely propagating u quark. To take into account the long-distance part one uses the expansion near the light-cone $x^2 = 0$ and introduces matrix elements of nonlocal quark-gluon operators between vacuum and real photon states schematically shown in Fig. 7.4e. The leading twist two contribution [84, 92] emerges from the expansion of the operator $\bar{u}(x) \sigma_{\alpha\beta} u(0)$ and the corresponding part of the matrix element in the external photon field can be parametrized as

$$\langle 0 | \bar{u}(x) \sigma_{\alpha\beta} u(0) | 0 \rangle_{F(q)} = e_u \langle \bar{u}u \rangle \int_0^1 du \varphi(u)_\gamma F_{\alpha\beta}(ux) . \tag{98}$$

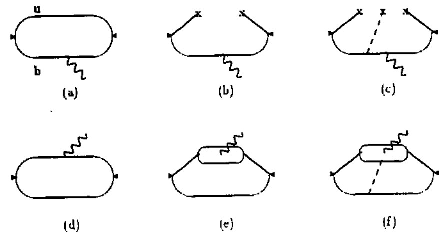

Fig. 7.4: Diagrams corresponding to OPE of the correlation function in (96): (a), (d) perturbative contributions, (b) quark, (c) quark-gluon condensate contributions; (e) emission of photon at large distances parametrized by photon wave function; (f) gluon correction to the diagram (e). Solid lines denote quarks, dashed lines -gluons, wavy line -photon, arrows - external currents. Crosses denote vacuum quark and gluon condensates, ovals in the diagrams (e) and (f) denote the photon light-cone wave function.

The path-ordered exponential gauge factors for both gluon and photon fields are not shown for brevity. The function φ_γ has the meaning of the photon wave function in terms of its quark-antiquark constituents and may be interpreted as the distribution in the fraction of light-cone momentum $q_0 + q_3$ of the photon carried by a quark. The asymptotic form of this wave function is known [16]:

$$\varphi_\gamma(u) = 6\chi u(1-u) . \tag{99}$$

The parameter χ normalizing this wave function is the so called induced quark condensate

$$\langle 0|\bar{q}\sigma_{\alpha\beta}q|0\rangle_F = e_q\langle\bar{q}q\rangle\chi F_{\alpha\beta} \tag{100}$$

[14, 15] where e_q is the quark electric charge, $\langle\bar{q}q\rangle$ is the quark condensate density. This parameter has physical meaning of magnetic susceptibility of the quark condensate. According to the analysis carried out in ref. [92], non-asymptotic effects in $\varphi(u)$ and higher twist 4 contributions to (98) are small, contrary to the case of the pion wave functions. This observation, as well as a rough estimate of the twist four contribution allow us to neglect all higher twist effects. Collecting all accounted contributions of the OPE we equate the obtained result for Π_μ with the hadronic representation using dispersion relations for the invariant amplitudes:

$$\Pi_{PC(PV)} = \frac{if_B m_B^2 A_{PC(PV)}}{m_b[m_{B^2} - (p+q)^2]} + \int_{s_0}^{\infty} ds \frac{\rho^h_{PC(PV)}(s,p^2)}{s-(p+q)^2} \tag{101}$$

where the first term represents the B-meson contribution. The second term parametrizing contribution of the higher states in B channel is estimated, as usual, with the help of quark-hadron duality. The final sum rule for the parity conserving amplitude after Borel transformation is

$$\begin{aligned} A_{PC} = \frac{m_b}{f_B m_B^2}\Bigg\{ &\int_\Delta^1 \frac{du}{u} \exp\left[\frac{m_B^2}{M^2} - \frac{m_b^2 - p^2(1-u)}{uM^2}\right] \Bigg[e_u\langle\bar{u}u\rangle\varphi(u) \\ &+ \frac{3m_b}{4\pi^2}\left((e_u - e_b)\frac{(m_b^2-p^2)(1-u)}{m_b^2 - p^2(1-u)} + e_b ln\left[\frac{m_b^2 - p^2(1-u)}{um_b^2}\right]\right)\Bigg] \\ &- \frac{e_b\langle\bar{u}u\rangle}{m_b^2 - p^2}\exp\left(\frac{m_B^2 - m_b^2}{M^2}\right)\Bigg\}, \end{aligned} \tag{102}$$

The parity violating amplitude has very similar sum rule. For the problem under investigation we need only the values of these amplitudes at $p^2 = m_\rho^2$ or at $p^2 = m_{K^*}^2$. The sum rules are valid in a much wider region of timelike variable p^2 , parametrically up to $O(m_b^2 - O(\mathrm{GeV}^2))$ and practically up to p^2=15 GeV^2. Independently, the sum rules similar to (102) were derived in [93].

The important point is that all parameters entering the r.h.s of eq. (102) are known since they also enter other QCD sum rules. The value of magnetic susceptibility was determined several times [94],[95] with essentially the same result $\chi = -4.4\mathrm{GeV}^{-2}$ at the normalization scale of 1 GeV.

As expected, roughly half of the values of the amplitudes $A_{PC,PV}$ comes from the contribution of the photon twist-two wave function. The contribution from the loop diagram with photon emission from the light quark is about the same and provides the second half of the total result. Both perturbative and nonperturbative photon emission from the heavy quark are negligibly small. The strong isospin violation (or in other words, dependence on the flavour of the spectator quark) which manifests itself in drastic difference of the amplitudes of charged and neutral modes, is one of the characteristic features of this mechanism in contrast to the short distance penguin amplitudes which are independent of the flavour of the spectator quark.

Our final prediction for the amplitudes defining the matrix element (95) for the WA decay $B \to \rho\gamma$ is then compared with the corresponding matrix element of magnetic penguin operator

$$\langle\rho \mid \frac{em_b}{16\pi^2}\bar{d}\sigma_{\mu\nu}(1+\gamma_5)bF^{\mu\nu} \mid B\rangle \tag{103}$$

estimated by the same method of light-cone sum rules in [90]. Our result amounts to about a 10% correction to the penguin mediated short-distance amplitude for $B^- \to \rho^-\gamma$ with uncertainty at the level of 50% from the values of the CKM matrix elements. This correction is comparable to the present accuracy of the leading matrix element (103). With more refined techniques, however, the long-distance contribution of weak annihilation should presumably not be neglected in predicting the branching ratio of $B^- \to \rho^-\gamma$ and the CP-violating asymmetries. Our conservative estimate of the overall accuracy is about 15-20 %. The $O(\alpha_s)$ corrections to the correlation function (96) will improve it.

Unlike in radiative B-decays, the effect considered here is crucial in the corresponding D-decays where the short distance penguin contributions are completely negligible. It is immediate then to convert eq. (102) and analogous sum rule for the second invariant amplitude into the sum rules for the hadronic matrix elements $\langle 0 \mid (\bar{d}(\bar{u})L^\mu c) \mid D^{+(0)}\rangle_\gamma$ determining the radiative decays $D \to \rho\gamma$. One has just to replace b with c , $\bar{B}$ with D in the sum rule and substitute corresponding parameters.

We predict the branching ratio for $D^0 \to \bar{K}^{*0}\gamma$ to be $1.5\cdot 10^{-4}(a_2/0.5)^2$ and for $D_s \to \rho^+\gamma$ to be $2.8\cdot 10^{-5}$. The observation of these decays is an interesting task for the next generation of experiments in charm physics.

Replacing the ρ current with the leptonic current, one can apply the developed method also to a very interesting decay $B \to l\nu_l\gamma$ where the photon emission removes the usual chirality suppression. From this calculation, the ratio between the purely leptonic decay modes with and without photon is predicted ([78]) :

$$R_B^\mu = \frac{\Gamma(B \to \mu\nu_\mu\gamma)}{\Gamma(B \to \mu\nu_\mu)} \simeq 11.5(180\mathrm{MeV}/f_B)^2 \tag{104}$$

This result is in agreement with the phenomenological estimates [96, 97], and underlines the importance of the radiative leptonic decays.

8 Conclusion

In conclusion, we list the main results obtained in the thesis:

1. A new method of QCD sum rules for three-point correlation functions is suggested, using double dispersion relations.

2. The QCD sum rules for radiative transitions in charmonium are obtained , and the gluon condensate contribution to these sum rules is calculated.

3. It is shown that double QCD sum rules for radiative transitions have an analog in nonrelativistic quantum mechanics.

4. The amplitudes of two-photon decays of charmonium are calculated in the sum rule approach with the account of gluon condensate contribution. The two-photon width of the η_b and the widths of heavy quarkonium decays to photon and light scalar or axial bosons are predicted in a model independent way.

5. A new way of the determination of gluon condensate density is suggested using the sum rules for two-photon widths of charmonium.

6. A new method of the calculation of virtual photon structure function in QCD is suggested. The first calculation of the hadronic part of the photon structure function in the intermediate region of Q^2 and x is carried out. The first estimate of the higher twist corrections to the photon structure function is obtained.

7. The first calculation of the charm contribution to the photon structure function is carried out with the account of the leading nonperturbative contribution.

8. A new method of calculating the moments of heavy quark-parton distribution in a heavy meson is suggested. Moments of the c and b quark distributions in D and B meson, respectively, are calculated. Dependence of the structure function on spin and parity of the heavy meson is predicted. Estimates of heavy quark fragmentation function moments are obtained in model independent way.

9. The method of estimating the intercept of Regge trajectory of heavy quarkonium states is suggested.

10. The first QCD calculation of the nucleon structure function $h_1(x)$ is done.

11. The quark part of the proton spin is expressed via induced condensates in external flavour-singlet axial field. The breaking of local OPE is found for correlator of two flavour-singlet axial currents. Strong mixing of axial singlet mesons is predicted.

12. Light-cone sum rules for the form factor $B \to \pi$, $D \to \pi$ are obtained at momentum transfer squared up to $m_{b,c}^2 - O(\text{ GeV}^2)$ and with twist 4 accuracy.

13. The method of deriving the light-cone sum rules for strong couplings of heavy mesons with pions is suggested and couplings $B^*B\pi$ and $D^*D\pi$ are estimated.

14. New model-independent method of calculation of long-distance effects in $B \to \rho\gamma$ decays is suggested where the photon light-cone wave function is essentially used.

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Appendix. The list of publications underlying this report

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Ա. Յու. Խոջամիրյան

ՔՔԴ ԳՈՒՄԱՐՆԵՐԻ ԿԱՆՈՆՆԵՐԸ ԾԱՆՐ ՔՎԱՐԿՆԵՐԻ ԵՎ ՖՈՏՈՆՆԵՐԻ ՀԵՏ ՏԵՂԻ ՈՒՆԵՑՈՂ ՊՐՈՑԵՍՆԵՐԻ ՀԱՄԱՐ

ԱՄՓՈՓՈՒՄ

Ատենախոսությունը նվիրված է տարրական մասնիկների ուժեղ փոխազդեցության տեսության՝ Քվանտային Քրոմոդինամիկայի (ՔՔԴ) հաշվարկային մեթոդների զարգացմանը և կիրառմանը: ՔՔԴ գումարների կանոնների մոտեցումը ընդհանրացվում է ծանր քվարկների և ֆոտոնների հետ տեղի ունեցող հադրոնային պրոցեսների դինամիկական բնութագրերը հաշվելու համար:

Ստացվել են ՔՔԴ գումարների կանոնները չարմոնիումի ռադիացիոն անցումների և ֆոտոնային տրոհումների համար, հաշվարկվել է գլյուոնային կոնդենսատի ներդրումը այդ գումարների կանոնների մեջ: Մշակված է գլյուոնային կոնդենսատի խտության որոշման նոր մեթոդ, հիմնված չարմոնիումի երկուֆոտոնային տրոհումների հավանականությունների համար ստացված գումարների կանոնի վրա:

Առաջարկված է ֆոտոնի ստրուկտուրային ֆունկցիայի հաշվարկման նոր մեթոդ, որը հիմնված է ՔՔԴ օպերատորային վերլուծման վրա: Գնահատված է հմայված հադրոնների ներդրումը ֆոտոնի ստրուկտուրային ֆունկցիայի մեջ:

Առաջարկվել և մշակվել է ծանր մեզոնների մեջ ծանր քվարկ-պարտոնների բաշխումները ստանալու նոր մեթոդ:

Պրոտոնի h_1 (x) ստրուկտուրային ֆունկցիան առաջին անգամ հաշվարկվել է ՔՔԴ -յում: Պրոտոնի սպինի քվարկային մասը արտահայտվել է ՔՔԴ վակուումը բնութագրող պարամետրների միջոցով:

Ստացված են $B \to \pi$ և $D \to \pi$ ֆորմֆակտորների համար ՔՔԴ գումարների կանոններ, որոնք հիմնված են լուսային կոնի մոտ կատարված օպերատորային վերլուծման վրա: Առաջարկված է ծանր մեզոնների և պիոնների ուժեղ կապի հաստատունների համար ՔՔԴ գումարների կանոնների մեթոդ:

Մշակված է $B \to \rho \gamma$ տրոհումների մեջ տեղի ունեցող երկար տարածությունների էֆեկտները գնահատելու նոր մոտեցում:

Ходжамирян Александр Юрьевич

ПРАВИЛА СУММ КХД ДЛЯ ПРОЦЕССОВ С ТЯЖЕЛЫМИ КВАРКАМИ И ФОТОНАМИ

АННОТАЦИЯ

Диссертация посвящена развитию и приложению вычислительных методов теории сильного взоимодействия элементарных частиц - квантовой хромодинамики (КХД). Подход правил сумм КХД обобщен для вычисления динамических характеристик адресных процессов с тяжелыми кварками и фотонами.

Получены правила сумм КХД для радиационных переходов и фотонных распадов чармония, вычислен вклад глюонного конденсата в эти правила сумм. Развит новый метод определения плотности глюонного конденсата, основанный на правиле сумм для вероятностей двухфотонных ширин чармония.

Предложен новый метод вычисления структурной функции фотона, основанный на операторном разложении. Получена оценка вклада очарованных адронов в эту структурную функцию.

Предложен и развит новый метод вычисления распределений тяжелых кварков-партонов в тяжелых мезонах.

В КХД впервые вычислена структурная функция протона h_1 (x). Кварковая часть спина протона выражена через параметры вакуума КХД.

Получены правила сумм КХД для формфакторов $B \to \pi$ и $D \to \pi$, основанные на операторном разложении вблизи светового конуса. Предложен метод правил сумм КХД для вычисления сильных констант связи тяжелых мезонов с пионами. Раработан новый подход к эффектам на больших расстояниях в распаде $B \to \rho\gamma$.